Continuous mode conversion of Lamb waves in CFRP plates

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Abstract

Online damage detection in thin walled light weight structures with Lamb waves is one common way to develop structural health monitoring (SHM) systems. Lamb waves occur in multiple modes, which can convert into each other under special conditions. The effect of mode conversion of Lamb waves is a well known phenomenon. Typically mode conversion takes place at structural changes regarding the geometry and material, e.g. damage, cracks, delaminations, etc and can be used as a criterion to get information about the health of the structure. However, experimentally we observed an unexpected continuous mode conversion (CMC) of Lamb waves in a multi-layer composite plate partially made of fabric material, which means, e.g., that the symmetric S₀-mode continuously converts into the A₀-mode without passing a discontinuity. This effect causes a considerably more complex wavefield and makes the detection and localization of failures more complicated. In this work, the new phenomenon of CMC is described and investigated experimentally as well as numerically.

(Some figures may appear in colour only in the online journal)

1. Introduction

In recent years, many research and application projects have dealt with the development of structural health monitoring (SHM) systems. One promising approach uses Lamb waves [1, 2]. Lamb waves are elastic ultrasonic waves which exist in thin plate-like structures. They have a low material and geometrical attenuation (1/√r) [3] and due to their small wavelength they interact even with low levels of damage [4]. Therefore, Lamb waves are interesting for monitoring of light weight CFRP (carbon fiber reinforced plastics) structures, e.g. airplanes, because it is possible to monitor large surfaces with a low number of sensors and actuators. However, Lamb waves have complex properties particularly in CFRP materials. They are dispersive and exist in at least two basic modes, a symmetric (S₀) and simultaneously an anti-symmetric (A₀) mode, and for higher frequencies higher order modes occur [5, 6]. Under special conditions the modes can convert into each other. For a conversion between the symmetric and anti-symmetric mode the wave has to travel through a discontinuity, which is not symmetric to the center plane of the wave and vice versa [7, 8]. Modes can only convert into other modes which exist at the same frequencies [9]. An example of mode conversion is given in figure 1. The S₀-mode interacts with a flat bottom hole and partially converts into an anti-symmetric A₀-mode.

However, additionally, we observed experimentally a continuous mode conversion (CMC) as Lamb waves travel through CFRP structures partially made of fabric material. In the following the effect of CMC is presented and some first ideas to explain this effect are given. The work is structured in the following parts. At first, the experimental setup is shown. Then, the effect of CMC is presented and the properties are explained. The proof of CMC is given and the identification of the modes is done using B-scans. Next, experimental and numerical models are used to derive a better explanation for this effect. Finally, the results and an outlook to further investigations are given.
2. Experimental setup

The experimental investigations are performed with the help of a 1D (PSV 300) as well as 3D (PSV 400 3D) scanning laser vibrometer from Polytec. Laser scanning vibrometry is widely used for the experimental investigation of Lamb waves [10–15]. The scanning laser vibrometer has a high spatial resolution and needs no coupling media. The laser vibrometry is based on the Doppler effect [16]. If an object moves at a velocity $v_{\text{object}}$ toward the laser beam, the frequency of the light is shifted by the Doppler shift. The frequency shift $\Delta f_D$ is measured. With equation (1) the velocity of the measured point can be calculated.

$$\Delta f_D = \frac{2f_{\text{laser}}v_{\text{object}}}{c} = \frac{2v_{\text{object}}}{\lambda_{\text{laser}}}, \quad (1)$$

The parameters $f_{\text{laser}}$, $v_{\text{object}}$ and $c$ are the frequency of the He–Ne Laser, the velocity of the reflection point and the light speed, respectively.

Table 1. Stacking sequence of the CFRP plate

<table>
<thead>
<tr>
<th>Layer</th>
<th>Orientation (deg)</th>
<th>Type</th>
<th>Layer thickness (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0/90</td>
<td>Twill fabric</td>
<td>0.4</td>
</tr>
<tr>
<td>2</td>
<td>+45</td>
<td>UD layer</td>
<td>0.25</td>
</tr>
<tr>
<td>3</td>
<td>−45</td>
<td>UD layer</td>
<td>0.25</td>
</tr>
<tr>
<td>4</td>
<td>0/90</td>
<td>Plain fabric</td>
<td>0.22</td>
</tr>
<tr>
<td>5</td>
<td>−45</td>
<td>UD layer</td>
<td>0.25</td>
</tr>
<tr>
<td>6</td>
<td>+45</td>
<td>UD layer</td>
<td>0.25</td>
</tr>
<tr>
<td>7</td>
<td>0/90</td>
<td>Twill fabric</td>
<td>0.4</td>
</tr>
</tbody>
</table>

Using three lasers the velocity components in the direction of a Cartesian coordinate system $(x_1, x_2, x_3)$ can be determined.

Figure 2 shows the experimental setup. The 3D laser vibrometer scans the top surface of a CFRP plate (1 m × 1 m × 2.02 mm), which is positioned on foam. The average distances between the lasers and the plate are 0.62 m or 0.94 m. The stacking sequence of the CFRP plate is given in table 1. The center layer is made of a plain fabric (see figure 10(a)) and the top and bottom surfaces are made of a twill fabric (see figure 10(b)), respectively.

The Lamb wave is excited using a piezoceramic actuator of 20 mm diameter and a thickness of $h = 1$ mm made from the material Marco FPM202. The actuator is attached at the center of the bottom surface of the plate with the aid of paraffin. Paraffin allows a reversible coupling between the actuator and the plate. The edges are prepared with silicon to reduce the amplitude of reflected waves. The top surface is coated by a retro-reflective layer to enhance the signal to noise ratio of the measurement signals.

A five-cycle sinus burst amplified by a NF-HSA-4011-amplifier is fed to the actuator. During the measurement the

sinus burst is repeated for each measurement point. The three lasers measure the velocity in the direction of the laser beams. With this information and the position of the laser devices the three velocity components of a point at the plate surface can be determined. After scanning each point a C-scan similar to figure 1 is created.

3. CMC

3.1. Description of the phenomenon

Figure 3 shows the out-of-plane velocity field of an undamaged CFRP plate for an excitation frequency of $f = 200$ kHz. The source of the wave is at the center of the bottom surface of the plate. The $x_1$-axis of the global coordinate system corresponds to the zero degree orientation of the CFRP plate. Two primary modes are excited by the piezoceramic source, the fast $S_0$-mode (long wavelength) and the slower $A_0$-mode (short wavelength). Due to their different velocities after some time the modes are clearly separated as shown in figure 3. The term primary means that both modes are excited directly by the source. Reflected or converted modes are named in a different manner. For all presented scans only the out-of-plane displacements are shown. The in-plane components measured by the 3D laser scanning vibrometer are used to get a better identification of the $S_0$-mode. Inside the $S_0$-mode other waves occur. These unexpected new waves are characterized by plane wavefronts being nearly parallel to each other. The orientation of the wavefront of the new mode depends on the region where the mode arises. In the bottom left part of figure 3 the different orientation between two wavefronts is shown. In addition, the new waves do not occur everywhere in the plate. For the excitation frequency 200 kHz the upper left and the lower right region do not show new modes. Nevertheless, these regions change if Lamb waves are excited by another frequency. Figure 4 compares two C-scans of the excitation frequencies of $f = 150$ and 250 kHz. The regions of appearance as well as their wavelength have changed. Nevertheless, the orientation of the mode has remained similar to before. The investigation of the given plate by applying several frequencies has shown that the new modes take place for the first time at a frequency of 138 kHz and appear at all investigated frequencies in the range of 138–350 kHz.

To figure out the type of the additional mode which arises in the $S_0$-wavefield the dispersion curves of the presented
CFRP plate are shown in figure 5. Lamb waves exhibit velocity dispersion and the dispersion curves illustrate the dependency between the frequency times thickness and the wave velocity. Furthermore, the existence of higher order modes and their cut off frequencies can be found. The curves in figure 5 are calculated with the semi-analytical finite element (SAFE) method, which has been proven to be very accurate for such CFRP plates [7]. The dispersion curves for the investigated plate show that for frequencies lower than 205 kHz only three modes exist, the A_0-, the SH_0- and the S_0-mode. Above this frequency a fourth mode appears, the A_1-mode. At frequencies higher than 280 kHz the symmetric S_1-mode occurs. In figures 3 and 4 the anti-symmetric A_0-mode has a shorter wavelength. Its wavefield is next to the source and the wavefront has a smaller diameter in comparison to the S_0-wavefield. The waves inside the S_0-mode have wavelengths similar to the A_0-mode. The three C-scan examples correlate to the values of the entries 0.3, 0.4 and 0.5 MHz mm in the dispersion curves in figure 5. Therefore, at the lowest frequency f = 150 kHz no higher order mode exists, which means that only a conversion from the symmetric mode into the anti-symmetric mode or the shear-horizontal mode can take place. However, the SH_0-mode has a significantly greater wavelength

\[ \lambda = \frac{c_p}{f} \]  

(2)
in comparison to the A_0-mode, because of its higher phase velocities c_p. For the lowest presented frequency f = 150 kHz it is assumed that the S_0-mode converts into an A_0-mode. As the A_0-mode is slower than the S_0-mode, the existence of the anti-symmetric mode inside the S_0-wavefield gives strong evidence for CMC.

Six identically produced CFRP plates have been investigated. All of them show an analog behavior. Therefore, random local damage inside the plate can be excluded as the reason for mode conversion.

As shown in figure 6(a), a strip of the Lamb wave C-scan is taken from a plate. The source is located at the left side. The primary modes are already separated from each other. In the space between both primary modes a number of oblique waves can be seen. These waves travel at the same velocity as the A_0-mode. Using the center line perpendicular to the wavefront of the primary modes the C-scan strip can be transformed into a B-scan. The time–amplitude data of each point of the center line of figure 6(a) are plotted side by side as illustrated in figure 6(b). The heights of the amplitudes at a specific time and position are visualized by the grayscale picture. Dark gray illustrates negative and bright gray positive amplitude values.

If a Lamb wave is excited by a burst signal, at least two primary groups of waves travel through the plate which may separate from each other. The groups propagate with different velocities. The B-scan displays the movement of these groups by oblique parallel lines. The inclinations of the lines correspond to the group velocities (see figure 5(b)) of the excited modes. The dominant lines with the highest amplitudes at a specific time and position are visualized by the grayscale picture. Dark gray illustrates negative and bright gray positive amplitude values.

In the following section, further arguments are gathered to identify the converted mode as A_0-mode. As the A_0-mode is slower than the S_0-mode, the existence of the anti-symmetric mode inside the S_0-wavefield gives strong evidence for CMC. Six identically produced CFRP plates have been investigated. All of them show an analog behavior. Therefore, random local damage inside the plate can be excluded as the reason for mode conversion.
Figure 7. B-scans for the figure 4. (a) 150 kHz. (b) 250 kHz.

Figure 8. Example of a mode conversion in a two-dimensional model of an aluminum plate. (a) Pure S<sub>0</sub>-mode. (b) Conversion of S<sub>0</sub>-in A<sub>0</sub>-mode.

The B-scans of the two other frequencies are plotted in figure 7. Figure 7(a) shows the B-scan for an excitation frequency of 150 kHz. The mode occurring in the S<sub>0</sub>-mode runs in the opposite direction compared to the primary A<sub>0</sub>-mode. The reversal point is about 175 kHz in the zero degree direction of the investigated CFRP plate. At this frequency nearly no mode conversion takes place in the whole plate. In figure 7(b) the B-scan for an excitation frequency of 250 kHz is shown. The converted mode travels in the direction of the S<sub>0</sub>-mode. Also at the top of the figure another mode can be seen. The gradient of the line corresponding to the mode is very small in comparison to the other two occurring modes. This mode can be identified as the A<sub>1</sub>-mode also arising at this excitation frequency.

4. Interpretation of CMC

4.1. Analysis of the problem

After presenting the phenomenon of CMC, we try to explain its appearance. The conversion between two modes occurs if a Lamb wave mode hits a discontinuity in such way that parts of the particle movement activate another mode. Typically mode conversion happens locally at holes, damage, etc. Figure 8 shows a two-dimensional example to explain mode conversion at a flat bottom hole. A symmetric mode is excited at the left edge and travels (figure 8(a)) through the plate. If the symmetric mode reaches the flat bottom hole a mode conversion takes place (figure 8(b)). The symmetric mode in isotropic material has an approximately constant in-plane displacement over the thickness of the plate. If the mode travels through the flat bottom hole the stiffness changes due to the reduction of the height. In the upper part of the plate the wave is reflected and in the lower part the mode travels further, which causes bending (figure 8(b)) of the plate. As the anti-symmetric mode is a bending mode, parts of the energy
of the symmetric mode are transferred to the anti-symmetric one and two modes exist.

Table 2 illustrates the primary displacements of the three basic modes in different directions for an isotropic material with an infinite extension in the \(x_2\)-direction (exemplarily shown in figure 8). A mode conversion takes place if the symmetric displacement of the \(S_0\)-mode in the \(x_3\)-direction causes an anti-symmetric displacement in the \(x_3\)-direction and vice versa.

In the following part we try to explain why CMC takes place in the presented CFRP plate. Figure 9 shows a strip of the Lamb wave C-scan with a length of \(l = 64\) mm. Behind the measured data a photograph of the plate surface is displayed. The texture of the twill fabric can be identified. The drawn lines are an extension of the fabric texture. The distance between two lines is 2 mm. It can be seen that the wavefront of the continuously converted mode corresponds to this direction. The top layer is made of twill fabric displayed in figure 10(b).

We assume that this texture causes the mode conversion. A plain fabric layer is also a part of the CFRP plate. The plain fabric has a texture like the twill fabric (figure 10(a)). Therefore, both fabrics are investigated. In the following two sections the Lamb wave propagation in both fabrics is studied. Experimental and numerical models are used for interpretation.

4.2. Plain fabric

4.2.1. Experimental investigation. First, a square two layer plain fabric plate (0.3 m \(\times\) 0.3 m \(\times\) 1 mm) is studied. The piezoceramic actuator is applied at the center of the rear surface of the plate. The top surface is measured by the scanning laser vibrometer, similar to the previously presented measurements. A five-pulse burst is used at different central frequencies between \(f = 100 \cdots 300\) kHz. Figure 11(a) shows the C-scan of the plain fabric plate for \(f = 200\) kHz. No mode conversion can be observed. The primary \(S_0\)-mode dominantly exists in the \(0^\circ\), \(90^\circ\), \(180^\circ\) and \(270^\circ\)-direction, whereas the \(A_0\)-wavefield is nearly circular. For lower or higher frequencies also no CMC occurs.

The B-scan in the \(x_1\)-direction, illustrated in figure 11(b), shows that no \(A_0\)-mode forms inside the \(S_0\)-mode lines. All lines corresponding to the \(A_0\)-mode start at the source. The experimental investigation of the two layer plain fabric plate does not give any evidence for CMC.

4.2.2. Numerical investigation. Although there is no CMC in the experiments a numerical analysis is performed. The material properties of the fibers as well as the matrix are given in table 3. As common, we assume transversal isotropic material properties for the fiber material [19]. Both the plain as well as the twill fabric model are calculated with the commercial finite element program ABAQUS. The Lamb waves are excited using a three-cycle Hann-window modulated sinusoidal point force in the \(x_3\)-direction at the center of the plate. All edges are assumed to be free.
4.3. Twill fabric

4.3.1. Experimental investigation. The experiments are accomplished at a single layer twill fabric plate (1 m x 1 m x 0.3 mm). As a result of the plate’s thickness the measurements are done several times after rotating the plate. We get the same results for all measurements, which means that the influence of bending of the plate under its own weight on the measurement results can be excluded. In figure 13(a) the C-scan for a frequency \( f = 20 \) kHz is plotted. The mode conversion occurs primarily in the 0°-, 90°-, 180°- and 270°-direction. Perpendicular and parallel to the texture of the twill fabric no mode conversion takes place.

Figure 12(a) shows the model to approximate a single plain fabric layer. In accordance with the previous definition the 0° direction of the fibers correlates with the \( x_1 \)-direction. The length and thickness are defined as \( l = 16 \text{ mm} \) and \( d = 2 \text{ mm} \). The cross section of the fiber is given with (2 mm x 2 mm). The square cell is copied and merged several times to create a complete plate. The fiber curvature of the model is neglected to reduce the computational cost.

The plain fabric plate is modeled by one layer with the size (0.46 m x 0.46 m x 2 mm). Similar to the experiments the Lamb waves are excited at several frequencies. In figure 12(b) a result of the calculations for the frequency \( f = 200 \) kHz is shown. A weak mode conversion in front of the primary \( A_0 \)-mode can be seen. The converted mode is oriented in the ±45°-direction corresponding to the texture of the plain fabric. The \( S_0 \)-mode for the plate with matrix material is not displayed, because the amplitudes are low in comparison to those of the anti-symmetric \( A_0 \)-mode.

Two additional models are applied to calculate a two layer plain fabric plate. It should be mentioned that the two layers can be arranged symmetrically or anti-symmetrically to the center plane. An anti-symmetric assembly of the two layers shows weak mode conversions, whereas the symmetric assembly shows no evidence of mode conversion. However, the reality is between both models. Therefore, if CMC occurs in a real two layer plain fabric plate the amplitudes of the converted \( A_0 \)-mode are quite small. Moreover, the material damping of the real structure is not considered in the numerical model which in reality also leads to a reduction of the amplitude of the converted \( A_0 \)-mode. Therefore, no converted \( A_0 \)-mode can be seen in the experimental investigation.
Figure 13. Experimental investigation of a one layer twill fabric for $f = 50$ kHz. (a) C-scan with the actuator in the center (100 mm $\times$ 100 mm). (b) B-scan.

4.3.2. Numerical investigation. Figure 14(a) shows the twill fabric model. The lengths and the thicknesses of the square twill fabric cell model are defined as $l = 32$ mm and $d = 2$ mm. The material properties of the fibers and matrix are given in table 3. The complete twill fabric plate (0.48 m $\times$ 0.48 m $\times$ 2 mm) is modeled with a single layer. Similar to the numerical plain fabric model the fiber curvature is neglected. The results of the numerical twill fabric model show the phenomenon of CMCs illustrated in figure 14(b). The main regions of occurrence of CMC are similar to the experiments. The mode conversion mainly takes place in the $0^\circ$, $90^\circ$, $180^\circ$, and $270^\circ$-direction. However, the orientation of the converted mode (between the black lines) differs from the experiments.

All numerical models, the plain fabric as well as the twill fabric models, illustrate the strong influence of the model assumptions on the results. In order to ensure quite accurate simulation results the applied linear finite elements should not be distorted. Otherwise the shear locking phenomenon takes place, which may reduce the accuracy considerably. However, unfortunately, the simulation of the real thin plate would require an enormous amount of degrees of freedom which cannot be handled by our existing advanced computer hardware yet. Consequently, in our simulation we used a greater plate thickness in comparison with the real plate, which is used in the experiments. Moreover, the modeling of the matrix–fiber arrangement has to be studied in detail.

5. Conclusions

The paper presents the phenomenon of continuous Lamb wave mode conversion in a CFRP plate. The effect is observed first in a multi-layer composite plate, where 50% of the whole plate is made of different types of fabrics. The investigated CFRP plate shows the effect of CMC first at around 138 kHz, which
is identified as a mode conversion from the $S_0$-mode to the $A_0$-mode. For this identification the dispersion curves as well as B-scans are used. The dispersion curves show that only three modes occur in the observed frequency region. The big differences of the phase velocity between the different modes allow the identification of the new mode as the $A_0$-mode. The B-scans have shown that for frequencies between 138 and 175 kHz the new $A_0$-mode travels opposite to the parent $S_0$-mode. Above 175 kHz both the parent and the converted $A_0$-mode travel in the same direction.

The experimental results of the scanning laser vibrometer have shown that the wavefield becomes considerably more complex in comparison to results where mode conversion is not present. Therefore, for SHM applications CMC has to be taken into account because the evaluation and the signal processing of the received signals become more complicated.

The analysis of the experimental data suggested that the fabrics in the investigated CFRP plates are responsible for the CMC. Therefore, experimental and numerical analyses of both types of fabrics (plain and twill) are performed. The developed finite element models are a first step for a better theoretical understanding of the phenomenon. The experimental investigation of the double layer plain fabric plate did not show a mode conversion. Nevertheless, in the one layer finite element models a weak mode conversion occurs. In the twill fabric plate the conversion arises for all investigated frequencies $f > 20$ kHz. The orientation of the converted $A_0$-mode is parallel to the texture of the twill fabric. The applied numerical model shows comparable properties. The new mode occurs mainly in the $0^\circ$, $90^\circ$, $180^\circ$ and $270^\circ$-direction.

The discrepancies between the experiments and the numerical results have many causes. The numerical models are a simplification of the real structure. The real material data and the real matrix–fiber arrangement are not known. Moreover, due to requirements of linear finite elements the numerical model uses a 2 mm thick plate, whereas the thickness of the experimentally investigated plate is 0.3 mm. The single layer twill fabric plate needs at least two million degrees of freedom for an accurate solution. When using linear finite elements one has to pay attention to the elements aspect ratio, because otherwise the shear locking effect takes place. To avoid critical aspect ratios highly refined meshes are needed, which result in several million finite elements and many more degrees of freedom. These models have to be solved in the time domain by applying very small time steps. At present such simulations are beyond our existing hardware resources. This is the reason why a thicker numerical plate model has been investigated numerically. However, the development of new higher order finite elements, which have shown the ability to overcome the existing limitations [20], is in progress. Further investigations have to be considered in the near future, where the modeling of the complete seven layer CFRP plate could be possible.

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**References**


