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Modeling of a fluid-loaded smart shell structure for active noise and vibration control using a coupled finite element–boundary element approach

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Abstract

A recently developed approach for the simulation and design of a fluid-loaded lightweight structure with surface-mounted piezoelectric actuators and sensors capable of actively reducing the sound radiation and the vibration is presented. The objective of this paper is to describe the theoretical background of the approach in which the FEM is applied to model the actively controlled shell structure. The FEM is also employed to model finite fluid domains around the shell structure as well as fluid domains that are partially or totally bounded by the structure. Boundary elements are used to characterize the unbounded acoustic pressure fields. The approach presented is based on the coupling of piezoelectric and acoustic finite elements with boundary elements. A coupled finite element–boundary element model is derived by introducing coupling conditions at the fluid–fluid and fluid–structure interfaces. Because of the possibility of using piezoelectric patches as actuators and sensors, feedback control algorithms can be implemented directly into the multi-coupled structural–acoustic approach to provide a closed-loop model for the design of active noise and vibration control. In order to demonstrate the applicability of the approach developed, a number of test simulations are carried out and the results are compared with experimental data. As a test case, a box-shaped shell structure with surface-mounted piezoelectric actuators and four sensors and an open rearward end is considered. A comparison between the measured values and those predicted by the coupled finite element–boundary element model shows a good agreement.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

In recent years, increasing attention has been paid to vibration and acoustic noise control in many industrial applications. The control of noise and vibration is essential in the design process of a product, since it contributes to comfort, efficiency and safety. There are two different approaches to achieve noise and vibration attenuation. On the one hand, there is the widely used passive approach. Passive control techniques mostly reduce vibration and sound emission of structures by applying additional damping materials. These methods are best suited for a frequency range above 1 kHz. A drawback of this traditional way can be an increase in weight due to the application of additional damping materials. It is also possible to improve the structural–acoustic properties by modifying the structural geometry. An overview of the developments in structural design optimization for passive noise control can be found in Marburg [1].

Active control techniques provide an alternative way to minimize unwanted structural vibration and noise that moves more and more into the field of vision for designers, since they avoid an increase in structural weight. A very useful control technique that was first introduced by Fuller [2] is active structural–acoustic control (ASAC). In this concept...
Actuators are directly attached to the shell structure in order to reduce sound radiation by changing the dynamic behavior of the structure. ASAC is similar to active vibration control (AVC) [3] since control forces are applied to the structure, but the goals are different. The concept of AVC is developed with the goal of reducing structural vibration instead of the acoustic response. ASAC also differs from active noise control (ANC) [4]. The purpose of ANC is to reduce the unwanted sound of a fluid–structure system by producing an anti-noise through loudspeakers. In ANC and ASAC systems piezoelectric ceramics are widely used as sensors and actuators, because they can easily be mounted onto the vibrating structure. Piezoelectric ceramics are available in several configurations such as stacks or thin patches. Active control techniques are usually employed in applications where the frequency range of interest is between 50 Hz and 1 kHz. Over the past few years, several researchers have studied different control strategies for ASAC. For example, Li and Zhao [5] investigated ASAC on a rectangular fluid-loaded plate with piezoelectric layers using a velocity feedback control algorithm. In this study, velocity feedback has been proven to be a robust and effective control strategy. Ruckman and Fuller [6] applied a feedforward control approach to reduce actively the acoustic radiation of a fluid-loaded spherical shell structure. This approach uses linear quadratic optimal control to minimize the total radiated power.

The development and industrial application of smart structural–acoustic systems for active noise and vibration control require efficient and reliable simulation tools. Virtual models are of particular interest in the design process of a product, since they enable the testing of several control strategies and they are required to determine optimal sensor and actuator locations. A simulation of an active structural–acoustic system includes the modeling of the mechanical structure, the piezoelectric actuators and sensors, the interior and exterior fluids as well as the control algorithm. When dealing with a fluid-loaded shell structure these subsystems cannot be modeled separately, because it is well known that the presence of a surrounding fluid strongly influences the vibration behavior of thin-walled structures. The subsystems are connected to an overall model by introducing coupling conditions that take into account the fluid–structure interaction and the fluid–fluid interaction.

The accurate modeling of active noise and vibration control has become a topic of wide interest. Wang et al [7] modeled analytically the controlled behavior of plate-like structures. The effect of piezoelectric actuation is introduced in this model through equivalent bending moments. Gu and Fuller [8] used an analytic approach to describe the control of fluid-loaded plates. Active control of fluid-loaded plates has also been investigated analytically by Lee and Park [9].

More complex problems cannot be solved analytically, and it is necessary to use numerical methods. Several numerical techniques such as the finite element method (FEM) and the boundary element method (BEM) are available to predict the behavior of active structural–acoustic systems. A virtual model established entirely on the basis of the FEM is suggested by Khan et al [10]. In this work, infinite acoustic elements are applied to construct a two-dimensional model for harmonic analysis. An extension of this work is given by Lefèvre and Gabbert [11]. They developed a three-dimensional time-domain model which is suitable to design an optimal linear quadratic regulator (LQR).

Only a few studies deal with the coupling of piezoelectric finite elements with acoustic boundary elements. In Kaljević and Saravanos [12] a coupled FE–BE approach for computing the steady-state response of acoustic cavities bounded by piezoelectric composite shell structures is presented. The FE–BE approach is performed by describing the piezoelectric structure with finite elements and the fluid medium with boundary elements. Lerch et al [13] proposed a combined FE–FE–BE scheme for the modeling of acoustic antennas. Here, finite elements are applied to predict the behavior of the piezoelectric structure as well as the acoustic near-field, and boundary elements are employed to describe the far-field.

In both approaches the FE–BE model and the FE–FE–BE model are applied only to analyze the interaction of the piezoelectric structure with the surrounding fluid, whereas the influence of control is not considered. Zhang et al [14] developed an approach for modeling active vibration isolation and sound radiation control of underwater structures by integrating different control algorithms into the FE–BE model. This approach utilizes the added mass concept to provide a closed-loop model for time-domain simulations. A drawback of this approach is the restriction to pure mechanical structures without piezoelectric actuators and sensors.

In order to successfully employ piezoelectric patches in active noise and vibration control, the coupled response of fluid-loaded smart shell structures needs to be accurately and reliably predicted. Therefore the present paper proposes a FE–FE–BE formulation in the frequency domain, which allows the modeling of arbitrary-shaped structural–acoustic systems including the employed control strategy. The construction of the coupled formulation consists of three parts. First, the piezoelectric shell structure as well as the enclosed and surrounding fluids have to be modeled. In the next step, the subsystems are coupled to obtain a multi-coupled system. Lastly, the employed control is implemented in the overall model.

In the present approach, the FEM is applied to model the shell structure as well as the piezoelectric actuators. The finite element selected for structural modeling is a standard eight-node layered Mindlin-type shell element with electromechanical properties [15]. The FEM is capable of analyzing shell structures with irregularly shaped geometries and complex boundary conditions in statics and dynamics including piezoelectric materials by applying linear coupled electromechanical constitutive equations. Finite elements are also employed to model finite fluid domains around the shell structure as well as fluid domains that are partially or totally bounded by the structure, such as cavities. Quadratic hexahedron elements with twenty nodes are used to discretize the finite fluid domains. The FE analysis of acoustic fields offers the possibility to take into account inhomogeneities inside the fluid. A disadvantage of FE models is the large number of degrees of freedom. Quadratic conforming
boundary elements with eight nodes are utilized to characterize the unbounded acoustic pressure field around the shell structure and the finite fluid domains. In contrast to FEM, the BEM automatically fulfills the Sommerfeld radiation condition and does not produce, as the FEM does, reflections at the far-field boundaries. For this reason the BEM allows numerical prediction of sound pressure fields in unbounded domains. The BEM is able to reduce the computational effort when dealing with unbounded problems, since only the boundary of the radiating domains has to be discretized. One drawback of the classical BEM is the resulting type of matrices, which are fully populated, non-symmetric and frequency-dependent.

A coupled FE–FE–BE approach is derived by introducing coupling conditions which describe the fluid–structure and fluid–fluid interaction. The coupling conditions are obtained by assuming kinematic and dynamic continuity of the structural and acoustic variables at the interfaces. After coupling the subsystems, either the unknown structural displacements or the acoustic pressures can be eliminated to establish a system of equations in terms of acoustic or structural variables only. For the present study a different procedure is employed, where both the structural and the acoustic variables are retained in the final system of equations.

The modeling of fluid-loaded smart shell structures requires not only the construction of an appropriate structural–acoustic model, but also consideration of the involved control. In simulations of active noise and vibration suppression, the influence of control can be taken into account simply by implementing the corresponding control algorithm in the structural–acoustic model. The present study makes use of velocity feedback control to demonstrate how a closed-loop model can be simulated [4, 14]. The velocity feedback algorithm generates forces which are aimed to minimize the vibration of the structure. The reduction of the sound radiation in the far-field is not directly influenced by the control. Consequently, in the far-field the sound pressure may not be suppressed as much as the vibration of the structure itself, which can be seen as a drawback of the velocity feedback control.

In order to check the accuracy and the quality of the proposed modeling approach, numerous test examples were studied in detail, and partially the numerical solutions were also compared with measurements. It could be observed that in general the numerical predictions are in very good agreement with the experimental results.

This paper is structured as follows. First, the theoretical background of the numerical approach is presented in detail. The applied finite elements and boundary elements for modeling the mechanical, electromechanical and fluid fields as well as their coupling is given. Then it is shown how the control can be included in the overall multi-field model in the frequency domain, which results in an overall closed-loop model. Finally, as a test example, a box-shaped shell structure is presented to evaluate the numerical results. The dynamic behavior of the cover plate of the box can be actively influenced by a group of surface-mounted piezoelectric patches. The experiments are performed in reality and virtually based on the proposed coupled FE–FE–BE modeling approach. The comparison of the measured results and the numerical predictions shows good agreement. Additionally, the experimental and the numerical results reveal that the radiated sound field and structural vibrations of the box-shaped shell structure are significantly reduced by means of velocity feedback control. The paper concludes with a summary and an outlook to ongoing activities.

2. Finite element and boundary element modeling

In this section, the FEM is applied to develop a plane piezoelectric composite shell element for the analysis of smart structures and a hexahedron element for the modeling of finite acoustic fluid domains. The BEM is used to derive a quadrilateral boundary element for simulating infinite fluid regions. All equations are described using a Cartesian $x_1, x_2, x_3$-coordinate system.

2.1. Finite element model of piezoelectric shell structures

In the following, the theoretical background of a FE formulation for analyzing laminated plates with integrated piezoelectric actuators and sensors is briefly discussed. The finite element selected for structural modeling is a simple piezoelectric composite Mindlin-type shell element with eight nodes (figure 1). It is assumed that the thickness of the layers is the same at each node. Additionally it is presumed that the modeled composite laminate plate consists of perfectly bonded layers and the bonds are infinitesimally thin as well as nonshear deformable. To describe the electromechanical properties of the element in terms of the natural coordinates $\xi_1$ and $\xi_2$, the following quadratic serendipity shape functions are used for the corner nodes

\[ N_i = \frac{1}{2}(1 + \xi_1 \xi_{1i})(1 + \xi_2 \xi_{2i})(1 + \xi_1 \xi_{1i} + \xi_2 \xi_{2i} - 1) \]  

and for typical mid-side nodes located at $\xi_{1i} = 0$ and $\xi_{2i} = \pm 1$

\[ N_i = \frac{1}{2}(1 - \xi_1^2)(1 + \xi_2 \xi_{2i}). \]  

The shell element has five degrees of freedom $u_1, u_2, u_3, \theta_{12}, \theta_{13}$ at each node for elastic behavior and there is one potential degree of freedom $\varphi$ per layer to model the piezoelectric effect. The normal rotation $\theta_{13}$ is considered to be zero. The strain–displacement relationships for the used plane shell element are based on the Mindlin first order shear deformation theory.

Assuming small deformations, the strain–displacement relationship reads

\[ \varepsilon = B_u u, \]  

where $\varepsilon$ is the linear strain vector and $u$ is the vector with the nodal displacements and rotations. The strain–displacement matrix $B_u$ is given by

\[ B_u = [ B_{u1}^T \cdots B_{u3}^T \cdots B_{u5}^T ], \]  

where $T$ denotes the transpose.
where the submatrix $B^i_w$, which is associated with the node $i$ of the shell element, has the form

$$
B^i_w = \begin{bmatrix}
\frac{\partial K}{\partial x_i} & 0 & 0 & x_3 \frac{\partial K}{\partial x_3} & 0 \\
0 & \frac{\partial K}{\partial x_1} & \frac{\partial K}{\partial x_2} & 0 & x_3 \frac{\partial K}{\partial x_3} \\
0 & 0 & \frac{\partial K}{\partial x_2} & 0 & -x_3 \frac{\partial K}{\partial x_3} \\
0 & 0 & \frac{\partial K}{\partial x_3} & N_i & 0
\end{bmatrix} \quad (5)
$$

The poling direction of the piezoelectric layers is assumed to be coincident with the thickness direction $x_3$, which means that the electric field acts only perpendicular to the layers. Moreover, the difference in the electric potential $\psi$ is supposed to be constant in each layer of the shell element. The electric field, which varies linearly through the thickness of a piezoelectric layer, causes an in-plane expansion or contraction. This behavior is called the transverse piezoelectric effect. In Marinković et al. [16] it is shown that these assumptions are accurate enough in thin structure applications. For modeling the electric field only one electric degree of freedom per layer has to be specified within the element. Thus the electric field-potential relation can be written in the simple scalar form

$$E_k = B_{\psi k} \psi_k, \quad (6)$$

with

$$B_{\psi k} = -\frac{1}{\kappa_k} \quad (7)$$

Here $E_k$ is the electric field, $\psi_k$ is the difference in the electric potential and $\kappa_k$ is the thickness of the $k$th piezoelectric layer of the shell element.

The coupled electromechanical behavior of piezoelectric materials in low voltage, strain and stress applications can be modeled with sufficient accuracy by means of linearized constitutive equations. In matrix form, the constitutive relations for a piezoelectric layer $k$ are defined as [17]

$$\sigma_k = Q_k \epsilon - \kappa_k E_k, \quad (8)$$

$$D_k = \varepsilon_k \epsilon - \kappa_k E_k, \quad (9)$$

where $\sigma_k$ denotes the stress vector and $D_k$ is the dielectric displacement in thickness direction. $Q_k$ and $\epsilon_k$ are the plane-stress reduced elastic stiffness and the piezoelectric matrices respectively. The coefficient $\kappa_k$ represents the plane-stress reduced dielectric permittivity of the $k$th piezoelectric layer.

The piezoelectric constitutive relations given above have to be used within the weak form of the mechanical equilibrium equations [18] to derive the electromechanical FE equations of a piezoelectric layer by applying a standard Galerkin procedure. After adding the local equations of all layers and elements to a global model, the resulting system of coupled algebraic equations can be expressed as

$$\left[ M_\phi \ 0 \right] \ddot{\phi} + \left[ C_{\phi u} \ 0 \right] \dot{\phi} + \left[ K_u \ K_{uw} \right] \phi = \left[ f_u \ f_u \right], \quad (10)$$

where $M_\phi$ is the mass matrix, $K_u$ is the stiffness matrix and $K_{uw}$ is the dielectric matrix of the discretized piezoelectric composite shell structure. The piezoelectric coupling arises in the piezoelectric coupling matrix $K_{uw}$. For convenience, a Rayleigh damping is introduced into the system of equation (10) assuming that the matrix $C_u$ is a linear combination of the matrix $K_u$ and $K_{uw}$. The external loads are stored in the mechanical load vector $f_u$ and in the electric load vector $f_{\psi}$. Applying the Fourier transform to the obtained system of equation (10) leads to an equivalent system of equations in the frequency domain

$$\left[ -\Omega^2 M_u + i\Omega C_u + K_u \right] \tilde{u}_{\psi} = \left[ f_u \ f_{\psi} \right], \quad (11)$$

where the vectors $\tilde{u}$ and $\tilde{\psi}$ represent the complex amplitudes of the structural displacements as well as rotations and electric potentials. Here, $\Omega$ is the excitation frequency and $i$ is the imaginary unit.

### 2.2. Finite element model of the acoustic fluid

Computational prediction of noise can be achieved by the FEM as well as BEM. When dealing with far-field problems the BEM is very efficient. On the other hand, the FEM is more suitable for bounded fluid regions. For this reason a classical FE formulation is employed to model the finite fluid domains around the smart shell structure as well as the fluid domains that are partially or totally bounded by the structure.

The development of acoustic FE elements to calculate time harmonic acoustic pressure waves in homogeneous fluids is based on the Helmholtz equation

$$\Delta \tilde{p} + k^2 \tilde{p} = 0, \quad (12)$$

where $\tilde{p}$ is the complex amplitude of the time harmonic pressure and $k$ is the wavenumber. In equation (12) $\Delta$ is the Laplacian operator. The wavenumber $k$ is given by

$$k = \frac{\Omega}{c}, \quad (13)$$

where $c$ is the speed of sound in the fluid. In the present study, a 20-node hexahedron element is chosen to discretize the fluid domain. The pressure amplitude $\tilde{p}$ is considered as the nodal degree of freedom in the finite element. To approximate the acoustic pressure within the hexahedron element, the following quadratic shape functions are used for the corner nodes

$$N_i = \frac{1}{8} (1 + \xi_1 \xi_1)(1 + \xi_2 \xi_2)(1 + \xi_3 \xi_3)(\xi_1 \xi_1 + \xi_2 \xi_2 + \xi_3 \xi_3 - 2)$$

and for typical mid-side nodes located at $\xi_1 = 0$, $\xi_2 = \pm 1$ and $\xi_3 = \pm 1$

$$N_i = \frac{1}{8} (1 - \xi_1^2)(1 + \xi_2 \xi_2)(1 + \xi_3 \xi_3).$$

As in structural mechanics, the Helmholtz equation can be cast in a weak form to derive mass-like and stiffness-like element matrices. Following a standard FE assembly procedure, the matrix equation of the discretized fluid domain becomes

$$(-\Omega^2 M_p + K_p)\tilde{\phi} = -i\rho_0 \Delta \tilde{\phi}_p, \quad (16)$$

with the acoustic mass matrix $M_p$, the acoustic stiffness matrix $K_p$ and the acoustic load vector $\tilde{\phi}_p$ due to the prescribed normal velocities $\vec{v}_n$. The variable $\rho_0$ stands for the density of the acoustic medium.
2.3. Boundary element model of the acoustic fluid

Boundary elements are utilized instead of infinite acoustic elements to characterize the unbounded acoustic pressure field around the shell structure and the finite fluid domains. Infinite elements do not fulfill exactly the Sommerfeld radiation condition, and do not perform sufficiently well when located in the near-field. Boundary elements, on the other hand, can be placed at any arbitrary position. Furthermore, boundary elements lead to a significantly smaller system of equations, because the unknown variables are introduced only on the boundary of the radiating domains. In the present paper, a quadrilateral boundary element with eight nodes is applied to discretize the boundary of the radiating domains. The acoustic pressure \( \tilde{p} \) and the normal velocity \( \tilde{v}_n \) are the nodal degrees of freedom. They are linked by the relationship [19]

\[
\frac{\partial \tilde{p}}{\partial n} = -i\rho_0 \Omega \tilde{v}_n, \tag{17}
\]

where \( n \) is the unit normal vector directed outwards from the fluid domain. The development of a BE formulation is based on the Helmholtz equation (12), which by means of the weighted residual method and Green’s identity can be transformed into the following integral equation

\[
\int_O \tilde{p} \frac{\partial g}{\partial n} dO + c_p \tilde{p} = -i\rho_0 \Omega \int_O g \tilde{v}_n dO. \tag{18}
\]

Here \( g \) is the fundamental solution of the Helmholtz equation (12) and \( c_p \) is a factor describing the surface angle of the source point located on the boundary of the radiating domain \( O \). To derive the BE formulation for the acoustic fluid, the standard boundary element discretization together with the collocation method are applied to the integral equation (18).

In the same way as in the FEM, the unknown variables are approximated by shape functions. To interpolate the acoustic pressure and the normal velocity within the boundary element, quadratic serendipity shape functions given in equations (1) and (2) are applied. The resulting direct BE matrix equation reads

\[
H\tilde{p} = -i\rho_0 \Omega G\tilde{v}_n, \tag{19}
\]

where \( H \) and \( G \) are the influence matrices and the vectors \( \tilde{p} \) and \( \tilde{v}_n \) are the nodal values of the acoustic pressure and the normal velocity. As mentioned before, the BEM only has to deal with a two-dimensional surface model. However, the influence matrices \( H \) and \( G \) are fully populated and have to be computed for each frequency \( \Omega \), since they are frequency-dependent.

3. Multi-structural-acoustic coupling

The dynamic behavior of lightweight structures with interior and exterior fluid loading is strongly affected by the interaction between the subsystems. Especially in the case of thin-walled structures with openings, fluid–structure and fluid–fluid interactions take place. The purpose of the following section is to present a FE–FE–BE approach to model the multi-coupling that occurs in systems with fluid–structure and fluid–fluid interfaces.

The approach is illustrated using the multi-coupled structural–acoustic system shown in figure 2. As illustrated, \( S \) is a flexible thin-walled piezoelectric structure that radiates sound into the neighboring fluid domains \( V_B \) and \( V_H \). The fluid domain \( V_H \) consists of two parts: an inner region that is substantially enclosed by the structure \( S \), and an outer region that includes sound fields around the openings and acoustic near-fields relevant for controller design purposes. \( V_B \) is an unbounded exterior fluid domain surrounding the shell structure \( S \) and the finite fluid region \( V_H \).

In the approach, the FEM is applied to model the flexible piezoelectric shell structure as well as the finite fluid domain \( V_H \). The FE modeling of sound fields allows the design of direct acoustic control, since the acoustic pressure can be provided as nodal degree of freedom in order to develop a closed-loop model. To predict the harmonic behavior of the unbounded exterior fluid domain \( V_B \) the BEM is applied. A coupled FE–FE–BE formulation is derived by introducing coupling conditions at the fluid–fluid interface \( O_{HB} \) and the fluid–structure interfaces \( O_{SH} \) and \( O_{SB} \). The coupling conditions are obtained by assuming kinematic and dynamic continuity of structural and acoustic variables at the interfaces. The coupling conditions are given by

\[
\begin{align*}
\tilde{p}_H &= \tilde{p}_B, & \tilde{v}_{nH} &= \tilde{v}_{nB} & \text{on } O_{HB}, \\
\tilde{p}_H &= \sigma_{un}, & \tilde{v}_{nH} &= i\Omega \tilde{u}_{n} & \text{on } O_{SH}, \\
\tilde{p}_B &= \sigma_{un}, & \tilde{v}_{nB} &= i\Omega \tilde{u}_{n} & \text{on } O_{SB}.
\end{align*}
\tag{20}
\]
At the fluid–fluid interface $O_{HB}$ the acoustic pressure and the normal particle velocity of the finite and the infinite fluid regions are equal. At the fluid–structure interfaces $O_{SH}$ and $O_{SB}$, the particles of the fluid and the boundary of the structure move together. As a result, the normal components of the particle velocity and of the first time derivative of the structural displacement are equal, and the acoustic pressure and the normal stress at the surface of the structure are in equilibrium. 

Using equation (11) and the interface relations (20), the FE formulation of the fluid-loaded piezoelectric shell structure $S$ can be written as

$$
\begin{bmatrix}
\mathbf{K}_a & \mathbf{K}_\mathrm{ap} \\
\mathbf{K}_{\mathrm{ap}}^\top & -\mathbf{K}_p
\end{bmatrix}
\begin{bmatrix}
\tilde{\mathbf{u}} \\
\tilde{\mathbf{p}}
\end{bmatrix}
= 
\begin{bmatrix}
\mathbf{f}_u + L_{SH} \tilde{\mathbf{p}}^S + L_{SB} \tilde{\mathbf{p}}^S \\
\mathbf{f}_p
\end{bmatrix},
$$

with

$$
\mathbf{K}_a = -\Omega^2 \mathbf{M}_a + i\Omega \mathbf{C}_a + \mathbf{K}_p.
$$

In equation (21) there are two additional load vectors describing the sound pressure that acts on the vibrating structure. The matrices $L_{SH}$ and $L_{SB}$ are coupling matrices including the shape functions of the shell element and the corresponding hexahedron and boundary element respectively.

The model of the finite fluid region $V_H$ is based on the equations (16) and (20). If the vector of the acoustic pressure $\tilde{\mathbf{p}}$ in equation (16) is split into the inner degrees of freedom $\mathbf{p}_V$ and degrees of freedom along the boundaries $O_{SB}$ and $O_{HB}$, the FE formulation of the finite fluid region $V_H$ can be written as

$$
\begin{bmatrix}
\mathbf{K}_{11}^p & \mathbf{K}_{12}^p & \mathbf{K}_{13}^p \\
\mathbf{K}_{21}^p & \mathbf{K}_{22}^p & \mathbf{K}_{23}^p \\
\mathbf{K}_{31}^p & \mathbf{K}_{32}^p & \mathbf{K}_{33}^p
\end{bmatrix}
\begin{bmatrix}
\mathbf{p}_V^S \\
\mathbf{p}_H^S \\
\mathbf{p}_H^B
\end{bmatrix}
= 
\begin{bmatrix}
\rho_0 \Omega^2 L_{SH}^T \tilde{\mathbf{u}} \\
0 \\
i\rho_0 \Omega L_{HB} \tilde{\mathbf{v}}^H_{nB}
\end{bmatrix},
$$

with

$$
\mathbf{K}_{ij}^p = -\Omega^2 \mathbf{M}_{ij}^p + \mathbf{K}_{ij}^p.
$$

Due to the load of the mechanical structure and the exterior fluid, there are two additional load vectors. The matrix $L_{HB}$ is a further coupling matrix, which contains the shape functions of the hexahedron and the corresponding boundary element.

In an analogous manner, the BE formulation (19) can be split into degrees of freedom along the boundary $O_{SB}$ and degrees of freedom along the boundary $O_{HB}$ to predict the behavior of the unbounded exterior fluid domain $V_B$. The resulting equation reads

$$
\begin{bmatrix}
\mathbf{H}_1 \mathbf{H}_2 \\
\mathbf{H}_1 \mathbf{H}_2
\end{bmatrix}
\begin{bmatrix}
\mathbf{p}_B^S \\
\mathbf{T}_{HB} \mathbf{p}_H^B
\end{bmatrix}
= 
-\mathbf{i} \rho_0 \Omega
\begin{bmatrix}
\mathbf{G}_{11} \mathbf{G}_{12} \\
\mathbf{G}_{21} \mathbf{G}_{22}
\end{bmatrix}
\begin{bmatrix}
\tilde{\mathbf{v}}^H_{SB} \tilde{\mathbf{u}} \\
\tilde{\mathbf{v}}^H_{nB}
\end{bmatrix},
$$

where $\mathbf{T}_{HB}$ is a transformation matrix between the nodal pressures of the hexahedron and the boundary element. The matrix $\mathbf{T}_{SB}$ transforms the nodal displacement of the shell element into the normal velocity of the boundary element.

Combining equations (21), (23) and (25), and moving all unknowns to the left-hand side, the following coupled FE–FE–BE matrix equation is obtained

$$
\begin{bmatrix}
\mathbf{K}_a & \mathbf{K}_{ap} & -\mathbf{L}_{SH} & 0 & 0 & -\mathbf{L}_{SB} & 0 \\
-\rho_0 \Omega^2 \mathbf{L}_{SH}^T & \mathbf{K}_{11}^p & \mathbf{K}_{12}^p & \mathbf{K}_{13}^p & 0 & 0 & 0 \\
0 & 0 & \mathbf{K}_{22}^p & \mathbf{K}_{23}^p & 0 & 0 & 0 \\
0 & 0 & \mathbf{K}_{32}^p & \mathbf{K}_{33}^p & 0 & 0 & 0 \\
-\rho_0 \Omega^2 \mathbf{G}_{11} \mathbf{T}_{SB} & 0 & 0 & 0 & \mathbf{H}_1^T \mathbf{T}_{HB} & \mathbf{H}_1^T & \mathbf{i} \rho_0 \mathbf{G}_{12} \\
-\rho_0 \Omega^2 \mathbf{G}_{21} \mathbf{T}_{SB} & 0 & 0 & 0 & \mathbf{H}_2^T \mathbf{T}_{HB} & \mathbf{H}_2^T & \mathbf{i} \rho_0 \mathbf{G}_{22}
\end{bmatrix}
\begin{bmatrix}
\tilde{\mathbf{u}} \\
\mathbf{p}_V^S \\
\mathbf{p}_H^S \\
\mathbf{p}_H^B \\
\tilde{\mathbf{v}}^H_{SB} \\
\tilde{\mathbf{v}}^H_{nB}
\end{bmatrix}
= 
\begin{bmatrix}
\mathbf{f}_u \\
\mathbf{f}_p \\
0 \\
0 \\
0 \\
0
\end{bmatrix}.
$$

It should be noted that throughout the whole approach conforming discretizations are assumed. In general, structural FE and acoustic BE meshes do not match at their interfaces. This may be due to different shape functions or different element sizes. To overcome the limitations of the classical node-to-node contact algorithms, Bernardi et al. [20] developed a segment-to-segment discretization technique called mortar method. In contrast to the node-to-node discretization, in the mortar method the continuity constraints are not enforced at discrete finite or boundary element nodes, but are formulated along the entire coupling boundary in a weak integral sense. The mortar method offers a universal coupling strategy that can be easily applied to the present system in order to obtain the required coupling matrices between the three subsystems independently of a given discretization.

4. Velocity feedback control

The analysis of fluid-loaded smart shell structures requires not only the design of an appropriate structural–acoustic model but has also to take into consideration the involved control. In numerical analyses, the influence of control can be taken into account simply by implementing the corresponding control algorithm in the coupled structural–acoustic model. The present study makes use of velocity feedback control to demonstrate how a closed-loop model can be obtained. Collocated piezoelectric actuators and sensors are utilized to form the closed-loop control system. The collocated design of an actuator/sensor pair guarantees control stability. In the considered feedback control system, the sensor output voltage is differentiated, amplified by a constant gain $g$ and directly fed back to the collocated actuator. Due to the feedback, the collocated actuator generates counteracting moments which suppress the vibrations and the resulting sound radiation of the shell structure. The velocity feedback control law of a collocated actuator/sensor pair reads in the frequency domain

$$
\tilde{\mathbf{v}}_a = i\Omega g \tilde{\mathbf{v}}_s,
$$

where $\tilde{\mathbf{v}}_s$ is the voltage taken from the piezoelectric sensor and $\tilde{\mathbf{v}}_a$ is voltage applied to the piezoelectric actuator.

Considering a fluid-loaded shell structure with multiple collocated actuator/sensor pairs, the vector of the electric potentials $\tilde{\phi}$ in equation (26) can be split into degrees of
freedom of the sensor layers \( \hat{\varphi}_s \) and the actuator layers \( \hat{\varphi}_a \). The system of equation (26) then becomes

\[
\begin{bmatrix}
    \tilde{K}_a & \tilde{K}_{aT} & \tilde{K}_{a0} & -L_{aH} & 0 & 0 & -L_{aB} & 0 \\
    K_{aT} & -K_0 & 0 & 0 & -L_{aB} & 0 & 0 & 0 \\
    K_{a0} & 0 & -K_0 & 0 & 0 & -L_{aB} & 0 & 0 \\
    -\rho_0 \Omega^2 L_{aH} & 0 & 0 & K_{0T} & K_{1T} & K_{2T} & K_{3T} & 0 \\
    0 & 0 & 0 & K_{0T} & K_{1T} & K_{2T} & K_{3T} & 0 \\
    0 & 0 & 0 & K_{0T} & K_{1T} & K_{2T} & K_{3T} & -i\omega_0 L_{aB} \\
    -\rho_0 \Omega^2 G_{1T} & 0 & 0 & 0 & H^{12}_{THB} & H^{11} & i\omega_0 G_{12} \\
    -\rho_0 \Omega^2 G_{2T} & 0 & 0 & 0 & H^{22}_{THB} & H^{21} & i\omega_0 G_{22}
\end{bmatrix}
\times \begin{bmatrix}
    \tilde{u} \\
    \tilde{v}_s \\
    \tilde{p}_s^x \\
    \tilde{p}_s^y \\
    \tilde{v}_s^x \\
    \tilde{v}_s^y \\
\end{bmatrix}
= \begin{bmatrix}
    \tilde{f}_a \\
    \tilde{f}_s \\
    \tilde{f}_a \\
    \tilde{f}_s \\
    \tilde{f}_a \\
    \tilde{f}_s
\end{bmatrix},
\] (28)

Here it is assumed that each actuator/sensor pair is modeled by a separate independent piezoelectric composite shell element with one potential degree of freedom for each actuator and the sensor layer respectively. As a result, the dielectric matrix \( K_p \) becomes diagonal [5]. In addition, it is presumed that for the sensor layer the applied charge \( \tilde{p}_s \) is zero. In equation (28), the subscript \( a \) refers to the actuator layers and the subscript \( s \) refers to the sensor layers.

The electric potential on the sensors can be obtained in terms of the structural displacements from equation (28) as follows

\[
\tilde{\varphi}_s = K_{pT}^{-1} K_{p0}^{-1} \tilde{u}.
\] (29)

In feedback control systems, the electric potential on the actuators is known from the applied control law. In such cases, the global system of equations can be expressed in terms of the structural and acoustic variables only. Thus equation (28) can be rewritten as

\[
\begin{bmatrix}
    \tilde{K}_a + \tilde{K}_{aT} + \tilde{K}_{a0} & -L_{aH} & 0 & 0 & -L_{aB} & 0 \\
    -\rho_0 \Omega^2 L_{aH} & K_{1T} & K_{2T} & K_{3T} & 0 & 0 \\
    0 & K_{1T} & K_{2T} & K_{3T} & 0 & 0 \\
    -\rho_0 \Omega^2 G_{1T} & 0 & 0 & H^{12}_{THB} & H^{11} & i\omega_0 G_{12} \\
    -\rho_0 \Omega^2 G_{2T} & 0 & 0 & H^{22}_{THB} & H^{21} & i\omega_0 G_{22}
\end{bmatrix}
\times \begin{bmatrix}
    \tilde{u} \\
    \tilde{p}_s^x \\
    \tilde{p}_s^y \\
    \tilde{v}_s^x \\
    \tilde{v}_s^y \\
\end{bmatrix}
= \begin{bmatrix}
    \tilde{f}_a \\
    \tilde{f}_s \\
    \tilde{f}_a \\
    \tilde{f}_s \\
    \tilde{f}_a
\end{bmatrix},
\] (30)

In equation (30), the known electric potentials on the actuators \( \tilde{\varphi}_a \) appear as external force. Using velocity feedback control, the control algorithm of the multiple independent local feedback loops can be written in the following vector-matrix notation

\[
\tilde{\varphi}_a = \omega G_{as} \hat{\varphi}_a.
\] (31)

The control matrix \( G_{as} \) in equation (31) accomplishes two things. It determines the gains within the local feedback loops and relates the potential degrees of freedom of the sensor layers to those of the corresponding actuator layers. With the control algorithm (31) the prescribed electric potentials \( \tilde{\varphi}_a \) can also be expressed in terms of the structural displacements \( \tilde{u} \), and thus all the electric degrees of freedom in equation (28) can be condensed. The condensed system takes the following form

\[
\begin{bmatrix}
    K_a & K_{aT} K_{a0}^{-1} & -L_{aH} & 0 & 0 & -L_{aB} & 0 \\
    -\rho_0 \Omega^2 L_{aH} & K_{1T} & K_{2T} & K_{3T} & 0 & 0 & 0 \\
    0 & K_{1T} & K_{2T} & K_{3T} & 0 & 0 & 0 \\
    -\rho_0 \Omega^2 G_{1T} & 0 & 0 & H^{12}_{THB} & H^{11} & i\omega_0 G_{12} & 0 \\
    -\rho_0 \Omega^2 G_{2T} & 0 & 0 & H^{22}_{THB} & H^{21} & i\omega_0 G_{22} & 0
\end{bmatrix}
\times \begin{bmatrix}
    \tilde{u} \\
    \tilde{p}_s^x \\
    \tilde{p}_s^y \\
    \tilde{v}_s^x \\
    \tilde{v}_s^y \\
\end{bmatrix}
= \begin{bmatrix}
    \tilde{f}_a \\
    \tilde{f}_s \\
    \tilde{f}_a \\
    \tilde{f}_s \\
    \tilde{f}_a
\end{bmatrix},
\] (32)

where \( C_{as} \) is the active damping matrix [17]

\[
C_{as} = K_{p0}^{-1} G_{as} K_{pT}^{-1} K_{p0}^{-1}.
\] (33)

The obtained system of equation (32) describes the controlled behavior of fluid-loaded smart shell structures. Due to the implementation of velocity feedback control, an additional damping term occurs on the left-hand side of equation (32). The feedback control forces generated by the feedback voltage increase actively the damping of the system. From equation (31) it is known that the intensity of the active damping depends only on the chosen feedback gains. Thus, by adjusting the feedback gains, the goal of controlling the vibration and the resulting sound radiation of the shell structure can be achieved.

A drawback of velocity feedback control is that the sound radiation of the shell structure is not controlled directly, since the control forces are generated to minimize the vibration of the structure. Due to the indirect control, sound pressure in the far-field may not be suppressed as much as the structural vibration. This is due to the fact that the applied control suppresses not only the vibration but also causes changes in the shape of the vibration modes. Consequently it is possible that the modified mode shapes produce a higher sound pressure at some points, if the radiated sound waves enhance each other. To overcome the problem, several ASAC strategies have been proposed, such as optimal positioning of the actuator/sensor pairs to minimize radiated sound power of the structure.

It should be noted that modeling a fluid-loaded smart shell structure with another feedback control strategy can be realized in an analogous way as it has been demonstrated for velocity feedback control.

5. Numerical studies

The purpose of this section is to demonstrate the validity of the proposed modeling approach. For this reason a number of test simulations are carried out and the results are compared with experimental data. Although the present formulation is applicable to any geometry such as curved shell structures, this paper utilizes a box-shaped shell structure with a partially enclosed acoustic cavity and an unbounded surrounding fluid domain. The box-shaped shell structure consists of a flexible plate, four rigid walls and an open side, where the inner and outer fluids interact. The flexible plate is clamped at all sides to the rigid walls, which are assumed to be acoustically hard surfaces. Eight piezoceramic patches are bonded to the top
and bottom surfaces of the flexible plate to form a set of four collocated actuator/sensor pairs for active vibration and noise suppression. A velocity feedback control algorithm is applied to relate the sensor voltage to the actuator voltage in four independent closed feedback loops. Figure 3 shows the layout of the considered fluid-loaded smart shell structure.

The results presented in this section are obtained with the geometrical and material parameters given in tables 1, 2 and 3. The center of the first collocated actuator/sensor pair is located at \( x_1 = 300 \) mm, \( x_2 = 300 \) mm, the second at \( x_1 = 300 \) mm, \( x_2 = 450 \) mm, the third at \( x_1 = 450 \) mm, \( x_2 = 300 \) mm and the fourth at \( x_1 = 450 \) mm, \( x_2 = 450 \) mm.

In the approach, the FEM is applied to model the acoustic cavity \( V_H \) and the shell structure \( S \) including the rigid walls, the flexible plate as well as the surface attached piezoelectric actuators and sensors. The BEM is employed to model the exterior sound field \( V_B \). A number of 404 standard finite shell elements and four piezoelectric composite shell elements are used to model the box-shaped structure (figure 4). The acoustic cavity is approximated by 832 finite hexahedron elements. For the unbounded exterior fluid, a conforming discretization with 508 boundary elements is applied. In all these elements, quadratic serendipity shape functions are employed. It should be mentioned that the experimental setup does not exactly fulfill the clamped boundary conditions of the plate. For this reason model updating has been carried out by applying partially clamped boundary conditions.

The box-shaped structure is excited by a harmonic force, and consequently, the structural and the acoustic responses are also harmonic. In the present study, a harmonic point force is applied perpendicular to the outer surface of the plate at \( x_1 = 500 \) mm and \( x_2 = 500 \) mm. The matrix equation (32) models the overall behavior of the box-shaped shell structure in the frequency domain. It can be used to determine the frequency response functions of the uncontrolled as well as the controlled system. For the comparison of the numerical results with the experiments (figure 5), frequency response functions between the excitation force and the plate displacement at \( x_1 = 100 \) mm, \( x_2 = 200 \) mm are computed. Additionally, frequency response functions for the acoustic pressure in the cavity at \( x_1 = 300 \) mm, \( x_2 = 650 \) mm and \( x_3 = 525 \) mm are calculated.

Figure 6 compares computed and measured structural and acoustic responses of the uncontrolled system. In the simulations a frequency range from 40 to 200 Hz is chosen to guarantee that several eigenfrequencies of the multi-coupled structural–acoustic system are within this band. To achieve a damping effect through velocity feedback control, a feedback gain of \( g = 0.05 \) s is selected.

**Table 1.** Geometrical parameters of the cavity and material properties of the inner and outer fluid.

<table>
<thead>
<tr>
<th>Dimensions</th>
<th>Speed of sound ( c ) (mm s(^{-1}))</th>
<th>Density ( \rho_0 ) (kg ( \cdot ) 10(^3) mm(^{-3}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>( l_1 \times l_2 \times l_3 ) (mm(^3))</td>
<td>340 000</td>
<td>1.29 \times 10(^{-12})</td>
</tr>
</tbody>
</table>

**Table 2.** Geometrical and material parameters of the aluminum plate.

<table>
<thead>
<tr>
<th>Dimensions ( l_1 \times l_2 \times l_3 ) (mm(^3))</th>
<th>Density ( \rho_s ) (kg ( \cdot ) 10(^3) mm(^{-3}))</th>
<th>Young’s modulus ( E_s ) (N mm(^{-2}))</th>
<th>Poisson’s ratio ( \nu_s ) (( -))</th>
</tr>
</thead>
<tbody>
<tr>
<td>600 \times 900 \times 4</td>
<td>70 000</td>
<td>70 000</td>
<td>0.34</td>
</tr>
</tbody>
</table>

Figure 3. Multi-coupled structural–acoustic system. (a) Front view, (b) rear view, (c) cross-section.

Figure 4. Conforming FE–FE–BE discretization of the shell structure as well as enclosed and surrounding fluid.
Figure 5. Photographs of the experimental setup.

Figure 6. Comparison of measured and calculated frequency response functions. (a) Displacement. (b) Sound pressure.

Table 3. Geometrical and material parameters of the piezoelectric patches PIC 151.

<table>
<thead>
<tr>
<th>Dimensions</th>
<th>Density $\rho_p$ (kg $\cdot$ $10^3$ mm$^{-3}$)</th>
<th>Young’s modulus $E_p$ (N mm$^{-2}$)</th>
<th>Poisson’s ratio $\nu_p$ (–)</th>
<th>Piezoelectric constant $e_{31/32}$ (N mV$^{-1}$ mm$^{-1}$)</th>
<th>Dielectric constant $\kappa_{33}$ (N mV$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l_{x1} \times l_{x2} \times l_{x3}$ (mm$^3$)</td>
<td>7.8 $\times$ $10^{-5}$</td>
<td>613 26</td>
<td>0.38</td>
<td>$-1.915 \times 10^{-5}$</td>
<td>$1.047 \times 10^{-14}$</td>
</tr>
</tbody>
</table>

A comparison of the measured values and those predicted by the coupled FE–FE–BE model shows good agreement. As shown in figure 6, the frequency range of interest includes the first five eigenfrequencies of the coupled system. An examination of the computed mode shapes reveals that the first, third, fourth and fifth mode can be interpreted as modes mainly caused by the shell structure. Due to the presence of the acoustic cavity and the exterior fluid, the in vacuo eigenfrequencies of the plate modes shift. The second peak in figure 6 denotes the first resonance frequency of the acoustic cavity. The computed deformed shape of the box-shaped shell structure at the first five eigenfrequencies is provided in figure 7.

From the resonance peaks it can be observed that the sound pressure in the acoustic cavity is related to the vibration of the plate. Only the fourth eigenmode of the coupled system does not influence the sound pressure at the measurement position. This is due to the position of the measurement point, which is placed close to the center of the acoustic cavity. At this position the radiated sound waves of the fourth plate mode shape cancel each other out.

In figures 8 and 9 the results of the controlled system are presented. Figure 8 compares calculated and measured displacement responses of the uncontrolled and the controlled plate and figure 9 presents sound pressure responses for the uncontrolled and controlled case. In both figures it can be observed that measured data and numerical predictions agree very well. Additionally, the results show that a significant damping effect is achieved at resonance regions due to the implementation of the velocity feedback control. According to
Figure 7. Computed deformed shapes of the box-shaped shell structure.

Figure 8. Uncontrolled and controlled frequency response of the plate displacement. (a) Simulation. (b) Experiment.

Figure 8, attenuation of the plate displacement at the resonance regions is about 53%. The same behavior can be seen in figure 9, where the response of the pressure amplitudes is reduced by about 41%.

As mentioned before, when the shell structure is excited by a harmonic force, the harmonically vibrating plate radiates sound into the enclosed and surrounding fluid. Thus the acoustic pressure in radiated sound fields oscillates at a single
frequency. For further comparison with the experiments, steady-state acoustic pressure fields of the uncontrolled and controlled system are computed by solving the system of equation (32). The following figures illustrate the computed sound pressure distribution over the cross-section of the shell structure. Simulated and measured contour plots are obtained for the case that the plate is excited by a harmonic force with an amplitude of 1 N and a frequency of 60 Hz.

In both figures it can be noted that the experimentally measured pressure fields show high consistency with the computed ones. Moreover, the figures reveal that for the uncontrolled and controlled case, the sound pressure level inside the acoustic cavity is significantly higher than outside. From the contour plots in figures 10 and 11 it can also be seen that due to the controller influence the sound pressure level is reduced by approximately 5 dB. The attenuation demonstrates that velocity feedback control reduces in the considered system the responses of acoustic pressure locally as well as globally.

From the good agreement of the results, it can be concluded that the proposed FE–FE–BE formulation allows the modeling of structural–acoustic systems in combination with control. The developed approach differs from previous approaches, which focused either on the electromechanical or the structural–acoustic modeling, in that it involves all subsystems. In addition, the new approach enables not only the modeling of velocity feedback control, but also of other feedback control strategies. Moreover, there are no particular restrictions on the shape of the lightweight structures as well as the geometry of the surrounding acoustic fluids.

The experimental reference solutions were obtained with hardware-in-the-loop experiments using a dSPACE controller board which determines the necessary control outputs for the actuators. A shaker was employed to excite the plate with a harmonic force and an accelerometer was applied to detect the plate vibration. To measure the acoustic pressure, a microphone was positioned inside the cavity. The sound pressure distribution was measured with the help of a uniformly distributed microphone array consisting of $3 \times 7$ microphones with a grid spacing of 100 mm. During the measurements, the microphone array was moved several times to cover the whole cross-section of the box-shaped shell structure. Experimental and numerical determination of the
pressure fields was carried out with the same feedback gain as for the frequency response functions.

During the tests the box-shaped shell structure was placed in an anechoic chamber with a lower cut-off frequency of 100 Hz, which ensures a free-field environment above this frequency. Since measurement of the acoustic fields was carried out with a harmonic excitation of 60 Hz, the measured pressure distribution outside the cavity was non-uniform. More exact measurements could be achieved if the radiating structure was placed in an anechoic chamber for low frequencies.

Experimental testing revealed that velocity feedback control is a more robust strategy than acceleration or displacement feedback as far as phase shifts are concerned. Phase shifts are the main reason why the performance of feedback controllers is limited in real systems. Phase shifts are primarily caused by the dynamic response of the installed sensors, actuators and filters as well as the dSPACE controller board. The attenuation achieved with the testing rig could be improved by compensating the phase lag using the dSPACE controller board to modify the response of the plant within the frequency range of interest.

6. Conclusions

In this paper, a coupled FE–FE–BE formulation has been presented to simulate a fluid-loaded smart lightweight structure with surface-mounted piezoelectric actuators and sensors. In this approach, the FEM is applied to model the piezoelectric shell structure. For structural modeling a plane piezoelectric composite Mindlin-type shell element with eight nodes is used. The FEM is also adopted to predict the behavior of the finite fluid domains around the shell structure as well as fluid domains that are partially or totally bounded by the structure. To discretize the finite fluid domains acoustic hexahedron elements are chosen. Conforming boundary elements are used instead of infinite acoustic elements to model the unbounded acoustic pressure field around the shell structure and the finite fluid domains. A procedure has been presented to obtain a FE–FE–BE formulation for modeling the interaction at the fluid–fluid and fluid–structure interfaces. The resulting system of equations is obtained in terms of displacements of the shell structure, electric potentials of the piezoelectric layers and acoustic pressures of the fluid domains. This model has been constructed particularly to simulate the active noise and vibration suppression in the frequency domain. For this reason, it has been shown how a control algorithm can be implemented into the multi-coupled structural–acoustic formulation to provide a closed-loop model. In the present study a velocity feedback control is used to demonstrate the implementation. In addition, collocated piezoelectric actuators and sensors are assumed to form a simple closed-loop control system. The approach developed is very useful in order to determine the optimal number of piezoelectric actuator/sensor pairs, the dimension of each pair and their locations on the structure. In addition, the approach developed is easy to implement in existing FE–BE codes, and there is also no restriction regarding the shape of the structure when using the proposed approach. The coupled FE–FE–BE formulation leads to a significant smaller fluid mesh compared to the pure FE model, since boundary elements can be used in the near-field. With infinite elements, which are the counterparts to boundary elements, the whole near-field has to be discretized, because infinite elements perform sufficiently well only when they are positioned in the far-field. In contrast to the indirect boundary element method (IBEM), which is also well suited to model fluid-loaded shell structures, the approach developed offers the possibility to model inhomogeneities inside finite fluid domains.

To demonstrate the applicability and validity of the developments, test simulations are carried out and the results are compared with measurements. As a test case, a box-shaped shell structure with surface-mounted piezoelectric actuators and four sensors and an open rearward end is considered. It is shown that the measured values and those predicted by the coupled FE–FE–BE model are in a good agreement. It can be concluded that the proposed approach is able to model the uncontrolled as well as controlled behavior of fluid-loaded smart shell structures with a good quality. The present
developments provide a solid basis to model and to design fluid-loaded piezoelectric devices for active noise and vibration suppression.

Furthermore, experimental and numerical results revealed that the radiated sound field and structural vibrations of the box-shaped shell structure were significantly reduced by means of the designed velocity feedback control. Efficiency of the control could be further improved by introducing phase-lag compensators and by changing the position of the sensor/actuator pairs. In relation to ongoing research projects, an experimental setup consisting of a stripped engine will be built in order to test the performance of the methods developed on complex industrial-like applications.

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References