This paper presents a study on dynamics modeling and control system design of a stewart platform using piezoelectric actuators, which has been developed at the Spacecraft Dynamics Control and Navigation Laboratory (SDCNLab), York University for vibration isolation. As an efficient mechanism of six degree-of-freedom (6-DOF), stewart platform is capable of isolating vibration of wide band range, which is of essence in vibration control of aerospace structures. In this paper, the dynamics model of piezoelectric stewart platform in cubic configuration is first studied. In order to achieve the vibration isolation using the platform, a PD feedback control law, as well as a force and acceleration feedback control law, derived from the kinetic relation between attitudes of platform and elongation of piezoelectric actuators are discussed. Finally, simulations are conducted and the results verify the effectiveness of vibration isolation controllers.

1. Introduction

With deep investigation to the space, recent years, there is increasing interest in the vibration isolation applied to aerospace missions. While researcher are dedicated in developing various space mission like “Terrestrial Planet Finder” of deep space exploration and “Lunar Laser Communications Demonstration” in space laser communication, disturbance from usage of mechanical devices as reaction wheels, gyroscopes, solar array drives and other devices, as well as from outside spacial disturbance like space shuttle docking, gradually become severe and significant problems must been considered. Diversified attempts are implemented to solve this problems, such as vibration isolation tab, miniature vibration isolation system, and application of stewart platform. As a popular parallel mechanism invented by Gough and summarized by stewart [1], stewart platforms have wide-range application such as fighter simulator, vehicle simulator, vibration table, space dock mechanism [2]. One of such systems called “Cubic Configuration” was invented by Z. Geng and has been used by Intelligent Automation Inc. (IAI) [3]. The cubic configuration is obtained by cutting a cube through surface diagonals to two planes as base plate and moving plate. The edges of the cube are the six
struts connecting the two plates as a platform. In such construction, the cubic configuration has several unique characteristics, making such stewart platform capable of decoupled control action, since adjacent legs are orthogonal to each other, as well as using the symmetry of cube to provide identical legs. All these features mentioned would be very useful in vibration isolation. Because stewart platform has multiple freedom and precise position accuracy while maintaining high force-weight ratio, it become a ideal device in dealing with the wide-band frequency vibration in space structure. To satisfied the needs of high accuracy while eliminate the disturbance, plenties of control law such as are decentralized integral force feedback control [4], nonlinear robust control [5], fuzzy control [6] are proposed.

In this paper, a piezoelectric stewart platform with a cubic configuration, which is developed at the Spacecraft Dynamics Control and Navigation Laboratory (SDCNLab), York University for vibration isolation, is explored and the dynamic model of the vibration isolation platform is established based on Newton-Euler approach. Based on dynamics model, a PD feedback control principle to isolate the vibration, as well as a force-acceleration feedback control principle is proposed. The numerical simulation results with both control methods to the vibration isolation platform are presented.

2. Dynamic Modeling

The piezoelectric stewart platform developed at SDCNLab is shown in Figure 1.

![Figure 1. 6-DOF stewart platform developed at SDCNLab, York University](image)

As a normal “Cubic Configuration”, basic parameters overall length $l$, the distance from the center of the plate to the point of intersection of two adjacent legs $r$ and the overall height of the platform $Z$ have the following relations:

$$
\frac{r}{l} = \sqrt{\frac{2}{3}}; \quad \frac{Z}{l} = \sqrt{\frac{1}{3}}
$$

(1)

To estimate the Jacobian matrix that defines the relationship between the elongation of the legs and the motion of the platform, let us consider the vectorial representations of the hexapod shown in Figure 2, where the notations are [7]:

- ${\{B\}}$: base plate, defined as inertial reference frame at center $o_i$ with a coordinate $o_i x_i y_i z_i$.
- ${\{P\}}$: payload plate, defined as reference frame at center $o_r$ with a coordinate $o_r x_r y_r z_r$.
- $\vec{q}_i$: the vector of leg $l_i$, ($i = 1, 2, 3, 4, 5, 6$).
- $\vec{p}_k$: the position of the extremity of leg $l_i$ in the payload plate, defined as vector ($k = 1, 2, 3$).
- $\vec{b}_j$: the position of the extremity of leg $l_i$ in the base plate, defined as vector ($j = 1, 2, 3$).
- $R$: the rotation matrix transfer reference frame to inertial frame.
- $\vec{Z}_0$: the vector connecting $o_i$ to $o_r$. 
As we can see in Figure 2, the relationship between inertial frame and reference frame is completely defined by \( \vec{Z}_0 \) and \( \vec{\theta} \), where \( \vec{\theta} \) is a 3×1 vector of angular displacement of \( P \). In the modeling, since angular movement is small, an assumption that the rotation matrix transfer reference frame to inertial frame is approximately equal to \( I(3) \) is adopted. Different of stewart platform used to motion simulation, the elongation of legs linked to a vibration isolation platform is much smaller, traditionally within dozes of microns, this assumption is reasonable and acceptable in accuracy. The Jacobian matrix \( J \) relates the elongation velocities of the legs \( q_i \) to the velocity vector \( \dot{\chi} \), where \( \vec{V} = \dot{\vec{Z}}_0 = ( \vec{V}_x \ \vec{V}_y \ \vec{V}_z ) \) is a 3×1 vector of angular velocity of \( P \) to \( B \). Based on the assumption above we have:

\[
\vec{\omega} = \dot{\vec{\theta}}
\]

(2)

The definition of Jacobian Matrix can be expressed as:

\[
\dot{q} = J \dot{\chi}
\]

(3)

The absolute velocity \( \vec{V}_k \) of the extremity of \( \vec{p}_k \) can be expressed as:

\[
\vec{V}_k = \dot{\vec{Z}}_0 + \dot{\vec{p}}_k = \vec{V} - \vec{p}_k \times \vec{\omega}
\]

(4)

Since the extremity of \( \vec{b}_j \) is fixed in the inertial frame, the elongation velocity of the leg \( \dot{q}_i \) can be calculated as follow:

\[
\dot{q}_i = \dot{\vec{q}}_i \cdot \vec{V}_k / |\dot{\vec{q}}_i|
\]

(5)

From Figure 2, we can get:

\[
\vec{q}_i = (\vec{Z}_0 - \vec{b}_j) + \vec{p}_k
\]

(6)

Substitute Eqs. (4, 6) into Eq. (5), we have:

\[
\dot{q}_i = \dot{\vec{q}}_i \cdot \vec{V}_k / |\dot{\vec{q}}_i| = \frac{1}{|\dot{\vec{q}}_i|} \left[ (\vec{Z}_0 - \vec{b}_j) + \vec{p}_k \right] \cdot \vec{V} - \frac{1}{|\dot{\vec{q}}_i|} \left[ (\vec{Z}_0 - \vec{b}_j) \cdot \vec{p}_k \times \vec{\omega} \right]
\]

(7)

Usually, \( \vec{Z}_0 \) and \( \vec{b}_j \) are expressed in \( B \), \( \vec{p}_k \) is expressed in \( p \). In these conditions, we can project Eq. (7) in the reference frame \( p \):

\[
\dot{q} = J \dot{\chi} = \begin{pmatrix}
\frac{1}{7} \left[ (\vec{Z}_0 - \vec{b}_j)^T R + \vec{p}_k^T R \right] - \frac{1}{7} (\vec{Z}_0 - \vec{b}_j)^T R \vec{p}_k \\
\end{pmatrix} \begin{pmatrix}
\vec{V} \\
\vec{\omega}
\end{pmatrix}
\]

(8)

where we have used the anti-symmetric matrix \( \vec{p}_k \) to express the cross product \( \vec{p}_k \times \vec{\omega} = \vec{p}_k \vec{\omega}, |\dot{\vec{q}}_i| = l \).
Then, the Jacobian matrix becomes:

\[
J = \begin{pmatrix}
\frac{1}{\tau}(\vec{Z}_0 - \vec{b}_j)^T I(3) + \vec{p}_k^T & \cdots \\
\vdots \\
-\frac{1}{\tau}(\vec{Z}_0 - \vec{b}_j)^T I(3) \hat{p}_k \\
\end{pmatrix}
\]  

(9)

From Newton’s equation and Euler’s equation, we have:

\[
M \ddot{\chi} = \begin{bmatrix} \vec{F} \\ \vec{T} \end{bmatrix}
\]  

(10)

From the virtual work principal, we have:

\[
f^T \delta q = \begin{bmatrix} \vec{F}^T \\ \vec{T}^T \end{bmatrix} \delta \chi
\]  

(11)

where \(f\) is a \(6 \times 1\) vector of force coming from the legs. Derived from Eq. (3), we can get \(\delta q = J \delta \chi\).

Taking Eq. (11) into consideration, we get:

\[
\begin{bmatrix} \vec{F} \\ \vec{T} \end{bmatrix} = J^T f
\]  

(12)

Assuming that the only stiffness exists in the stewart platform is the axial stiffness of the leg, we have:

\[
f = -kq - c\dot{q}
\]  

(13)

where \(c\) is damping coefficient. Eq. (13) can be evolved to:

\[
f = -kJ\chi - cJ\dot{\chi}
\]  

(14)

Substitute Eqs. (12, 14) into Eq. (10), we have:

\[
M \ddot{\chi} + J^T cJ \dot{\chi} + J^T kJ \chi = 0
\]  

(15)

To get the expression of \(M\), consider the estimation of the kinetic energy \(T\) of payload platform (mobile platform) expressed in the frame \(P\):

\[
T = \frac{1}{2} m \cdot I(3) V_c^T V_c + \frac{1}{2} \omega^T I \omega
\]  

(16)

where \(m\) is the mass of mobile plate, \(V_c\) is the translational velocity vector of mass center of the mobile platform. \(I\) is the moment of inertia of the payload plate in reference frame \(P\), we get:

\[
V_c = \begin{pmatrix}
V_x + \omega_y Z_c - \omega_z Y_c \\
V_y + \omega_z X_c - \omega_x Z_c \\
V_z + \omega_x Y_c - \omega_y X_c
\end{pmatrix}
\]  

(17)

where \((X_c, Y_c, Z_c)^T\) is the vector of the mass center of the mobile plate in reference frame \(P\). In a general form, the kinetic energy can be calculated from the equation:

\[
T = \frac{1}{2} \chi^T M \dot{\chi}
\]  

(18)
Substitute Eqs. (17, 18) into Eq. (16), we have $M$ in reference frame $P$:

$$
M = m \begin{bmatrix}
1 & 1 & Z_c & -Y_c \\
1 & Y_c & -X_c & -Z_c \\
-Z_c & Y_c & Z_c^2 + Y_c^2 & -2X_cY_c \\
Z_c & -X_c & -2X_cZ_c & Y_c^2 + X_c^2
\end{bmatrix} + \begin{bmatrix}
0 & 0 \\
0 & I
\end{bmatrix}
$$

(19)

Rewrite Eq. (15) as follows:

$$
M\ddot{\chi} + C\dot{\chi} + K\chi = 0
$$

(20)

where: $C = J^T cJ$, $K = J^T kJ$.

Specifically, if the vector of mass center of the mobile plate is small, and is ignorable, we have:

$$
M = \begin{bmatrix}
m \cdot I(3) & 0 \\
0 & I
\end{bmatrix}
$$

(21)

Basically, the systemic damping of a stewart platform is small and ignorable, Eq. (20) can be written as:

$$
M\ddot{\chi} + K\chi = 0
$$

(22)

Specifically, in a basement vibration situation, the base plate is not still, assuming have a movement $\chi_B$, from equation Eqs. (3, 13), still ignoring the damping, we have:

$$
M\ddot{\chi} + K\chi = K\chi_B
$$

(23)

Generally, if there is an external forces and moments $F_{ext}$ acting on the payload platform, Eq. (23) becomes:

$$
M\ddot{\chi} + K\chi = K\chi_B + F_{ext} = F_g
$$

(24)

where the vector $F_g$ represent the grand forces and moments acting on the payload platform in a coordinate system consistent with $\chi$, either from external forces and moments or from base plate movement.

### 3. Control strategy

#### 3.1 PD Feedback Control

We can see from Eq. (22), the corner frequencies of the system are determined by the values of $M$ and $K$. To achieve the good passive, low-pass filter vibration isolation ability, it should be ensured that $K$ is as small as possible and $M$ is as large as possible. However, it is impossible as we take the carrying capacity and the size of the platform into consideration. In order to solve this contradicts with its load capacity and dimensions of stewart platform, an active control strategy must be adopted.

In our case, we build in each of the six legs with a piezo actuator. Considering Eq. (12) and Eq. (22), the governing equation of motion evolves to:

$$
M\ddot{\chi} + J^T k\delta = F_g
$$

(25)

where $\delta = \begin{bmatrix} \delta_1 & \delta_2 & \delta_3 & \delta_4 & \delta_5 & \delta_6 \end{bmatrix}$ is the vector of the 6 active control displacements of the piezo actuators, $k\delta$ is control force produced by actuator. Using the PD Feedback control strategy, the control law can be written as follow:

$$
k\delta = -K_p\chi - K_d\dot{\chi}
$$

(26)

Substitute Eq. (26) into Eq. (25), we have:

$$
M\ddot{\chi} + J^T K_d\dot{\chi} + (K + J^T K_p)\chi = F_g
$$

(27)
3.2 Force and Acceleration Feedback Control

Other than using $\chi$ and $\dot{\chi}$ as control input, we located force sensor in each leg of the platform and collocated with the piezo actuator to measure the axial force resulting in the leg. The sensor output equation becomes:

$$y = k(q - \delta)$$  \hspace{1cm} (28)

where $y = \begin{pmatrix} y_1 & y_2 & y_3 & y_4 & y_5 & y_6 \end{pmatrix}^T$ is the 6 force sensors outputs. Using the force feedback with constant gain $g$, the control law can be written as:

$$k\delta = gy - K_d\dot{\chi}$$  \hspace{1cm} (29)

Consequently, the closed-loop equation of motion becomes:

$$M\ddot{\chi} + J^T K_{d} + g\dot{\chi} + K_1 + g\chi = F_g$$  \hspace{1cm} (30)

4. Simulation and Analysis

To transform Eq. (22) into modal coordinates, one may substitute by $\chi = \Phi q$ and, assuming that the mode shapes are normalized according to $\Phi^T M \Phi = I$ and that $\Phi^T K \Phi = \omega_n^2$. Consequently, Eqs. (27, 30) can be transformed to:

$$\ddot{q} + \Phi^T J^T K_d \Phi \dot{q} + \Phi^T (K + J^T K_p) \Phi q = \Phi^T F_g$$  \hspace{1cm} (31)

$$\ddot{q} + \Phi^T J^T K_d \Phi \dot{q} + \Phi^T K \Phi q = \Phi^T F_g$$  \hspace{1cm} (32)

Accordingly, if we want to change the controlled closed looped system frequency and damping to $\omega_d$, $\zeta_d$, the PD Feedback control parameters in Eq. (31) can be calculated as follows:

$$\Phi^T (K + J^T K_p) \Phi = \omega_d^2$$  \hspace{1cm} (33)

$$\Phi^T J^T K_d \Phi = 2\zeta_d\omega_d$$  \hspace{1cm} (34)

The force and acceleration feedback control parameters in Eq. (32) can be calculated as follows:

$$\Phi^T K \Phi = \omega_d^2$$  \hspace{1cm} (35)

$$\Phi^T J^T K_d \Phi = 2\zeta_d\omega_d$$  \hspace{1cm} (36)

From Eqs. (31, 33, 34) or Eqs. (32, 35, 36), we have:

$$\frac{|\chi|}{|F_g/\omega_n^2|} = \frac{1}{\sqrt{(r_1^2 - r_2^2)^2 + 4\zeta_d^2 r_1 r_2}}$$  \hspace{1cm} (37)

where, $r_1 = \omega_d/\omega_n$, $r_2 = \theta/\omega_n$, $\theta$ is the corner frequency of grand forces and moments. Especially, in a basement vibration isolation situation, we have:

$$\frac{|\chi_B|}{|\chi|} = \frac{1}{\sqrt{(r_1^2 - r_2^2)^2 + 4\zeta_d^2 r_1 r_2}}$$  \hspace{1cm} (38)
Based on Eq. (38), we can draw a transmissibility picture as Figure 3. From the picture on the left, we know that there is no attenuation effect without active control \((r_1 = 1 \text{ and } \zeta = 0)\), if the corner frequency of grand forces and moments is lower than \(\sqrt{2}\omega_n\). While adding an active control to achieve the desired \(r_1\) and \(\zeta\), we can get an attenuation effect as expected even though the corner frequency of grand forces and moments is lower than \(\sqrt{2}\omega_n\), which can be observed obviously in the picture right.

According to the structure of the piezoelectric stewart platform developed at SDCNLab, the parameters are: \(l = 0.202\) m, \(k = 1 \times 10^7\) N/m, \(m = 80\) kg. Supposing there is basement disturbance \(\chi_B = 1 \times 10^{-6}\sin(2\pi \cdot 5)\). The designed parameters in the simulation are: \(\zeta_d = 0.2, r_1 = 5\). The simulation results are demonstrated in Figure 4 and Figure 5. The following results are obviously, which include: when disturbance frequency is far below the system corner frequencies, the attenuation is very small. With an active control, the stewart platform can achieve an expected vibration isolation effect even if disturbance frequency is far below the system corner frequencies. Based on Eqs. (33,34) or Eqs. (35, 36), both active control strategies can realize the vibration isolation effect. In fact, either PD feedback control or force and acceleration feedback control strategy have the same simulation results as illustrated in Figure 4 and Figure 5.
5. CONCLUSION

This paper summarizes the work conducted at SDCNLab, York University in vibration isolation using a cubic Stewart platform. First, the dynamic model of the Stewart platform is established utilizing Newton-Euler approach. Then based on the developed model, two active control strategies to isolate the vibrations are presented. Analysis and simulation indicated that the vibration isolation active control using piezoelectric actuators can achieve well-expected vibration isolation effect even if the disturbance frequency is far below the system corner frequencies, which is a main defect of a passive vibration isolation approach.

REFERENCES


