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MODELLING OF THE PLASTIC ANISOTROPY IN SHEET METALS ON DIFFERENT SCALES

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Abstract *In the first part of the paper, two different texture-dependent material models based on the Taylor assumption are discussed and applied to the simulation of deep drawing operations of aluminum. Special emphasis is given to the calibration of anisotropy which is predicted by these models in their standard forms. In the second part of the paper, the quadratic yield condition suggested by v. Mises and Hill is reconsidered. A micromechanical interpretation of the fourth-order anisotropy tensor in terms of texture coefficients is given.*

1 Introduction

This paper is divided into two parts. In the first part (Chapter 2), two different texture-dependent material models based on the Taylor assumption are discussed and applied to the simulation of deep drawing operations of aluminum. From the numerical point of view, large-scale FE computations based on the Taylor model are very time-intensive and storage-consuming if the crystallographic texture is approximated by several hundred discrete crystals. Furthermore, the Taylor model in its standard form, which is based on discrete crystal orientations, has the disadvantage that the anisotropy is significantly overestimated if only a small number of crystal orientations is used. Here this overestimation of anisotropy is quantitatively analysed, and two Taylor-type models which lead to a reduction of the sharpness of the crystallite orientation distribution function (codf) are suggested. One model is an elastic-viscoplastic Taylor model based on discrete orientations. The sharpness is reduced by introducing an isotropic background texture by means of an additional isotropic material law. The other model is a rigid-viscoplastic one which uses continuous model functions on the orientation space. This model allows for a direct incorporation of the scattering around an ideal texture component since the model contains the half-width as a microstructural parameter which can be biased.

In the second part of the paper (Chapter 3) the quadratic yield condition suggested by v. Mises [25] and Hill [14] is reconsidered. A micromechanical interpretation of the fourth-order anisotropy tensor in terms of texture coefficients [6, 7] is given. For the special case of an orthorhombic sample symmetry and a plane stress state, it is shown

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that the parameters of the v. Mises-Hill model can be identified by the texture of the material. This identification additionally requires the measurement of two mechanical quantities.

2 Texture Based Micro-Macro-Models

Ductile polycrystalline metals subjected to forming processes usually show anisotropic microstructures. A typical feature of an anisotropic microstructure is that the distributions of the grain orientation and the grain shape are direction-dependent (crystallographic and morphological texture). These anisotropies induce direction-dependent yield loci and contraction ratios. Furthermore, the development of ears during deep drawing processes is commonly observed. Due to the prior processing, the anisotropy may already exist before the forming operation, but can also evolve by the deformation process. In order to perform reliable forming simulations, micromechanically based material models offer the opportunity to incorporate microstructural information directly into the material model, and, thus, to put the modeling on a physically sound basis. Taylor type polycrystal models [24, 18, 19, 10, 21] or self-consistent schemes, see e.g. [22], belong to this class of micromechanically based material models. Although computationally much more expensive than phenomenological models, they are nowadays more and more used in the integration points of finite elements in order to bridge the gap between the grain-scale and the macro-scale.

If the two-point statistics of crystal orientations is isotropic, then the codf represents the dominant aspect of the microstructure. The evolution of the codf can be modeled most easily by Taylor type models. In the present work two Taylor type models are used. One model is an elastic-viscoplastic model based on discrete crystal orientations (DT - discrete Taylor model) [9]. The macroscopic Kirchhoff stress is given by a superposition of the single crystal stresses. The model is enhanced by an isotropic constitutive equation modeling the isotropic part of the texture (DT(I) - discrete Taylor model with isotropic background texture) [9]. More precisely, the DT model is modified by decomposing the stress tensor into two parts. One part describes the isotropic effective viscoplastic behavior due to a random texture. The other part results from a the superposition of the crystal stresses. Consequently, we have two types of volume fractions. One type corresponds to the single crystals, the other one describes how isotropic the microstructure is. The overestimation of anisotropy can be avoided by adapting the isotropic volume fraction.

The second model is an (elastic-)rigid-viscoplastic model based on continuous model functions on the orientation space (CT - continuous Taylor model) [8, 9]. The applied Mises-Fisher model functions [20, 12] permit an explicit modeling of the scattering around texture components. The Mises-Fisher distribution is a central distribution. The scattering of a texture component can be described by a half-width value b. In contrast to the model of [23], the CT model directly incorporates this parameter for the calculation of the macroscopic stresses [8, 9].

Both material models have been implemented into the finite element code ABAQUS [13] using the interface UMAT, and are applied to the simulation of deep drawing processes. Fig. 1 shows the experimental and the simulated earing profiles for the DT and

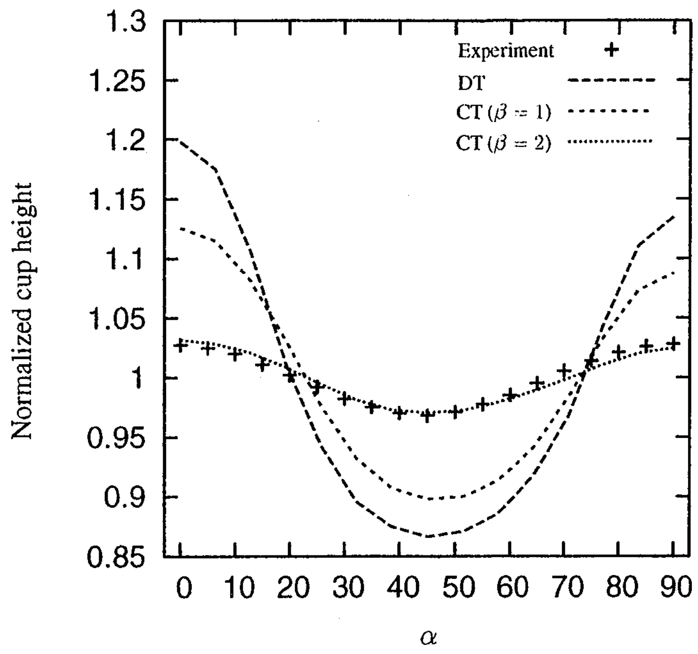

Figure 1: Comparison of the earing profile calculated by the DT and the CT model with experimental data [11]

the CT model. The starting texture for both models has been calculated from the experimental codf-section given in [11]. It can be seen that the discrete model drastically overestimates the earing height. This means that the anisotropy is overestimated. The continuous model also overestimates the earing height if the half-width is chosen in order to fit the codf section. If the half-width b is enlarged by a factor β of 2 then the continuous model rather accurately predicts the earing profile. Thus, the modification of the half-width allows to correct the predictions of the Taylor model which inherently overestimates the sharpness of the texture.

The predictions of the DT(I) model is analysed in the case of a pure cube texture. In Fig. 2 the predicted earing profiles are shown for the DT(I) and the CT model. In the case of the discrete model, the volume fraction of the isotropic part has been varied in the range of 30% - 90%. In case of the continuous model, the half-width has been varied in the range of 15° - 60°. It can be seen that the isotropic volume fraction of 30% corresponds approximately to a half-width of 15°. A volume fraction of 50% corresponds to a half-width of 30°. As a thumb rule, for half-width values larger than 10% one can use the fact that the isotropic volume fraction in the DT model is approximately given by $(10+240b/\pi)\%$ (b in rad). If one takes into consideration that even b should be increased by a factor of $2\ldots3$, then one has a rough estimate of the isotropic volume fraction in the DT model directly based on the codf. However, this estimate depends on the number of crystals involved in the DT model. The estimate given here can be considered as an upper bound. If more discrete orientations are used, the isotropic volume fraction should be smaller. Since for a small number of crystal orientations the discrete model is computational less expensive, this modification of the discrete Taylor model seems to be

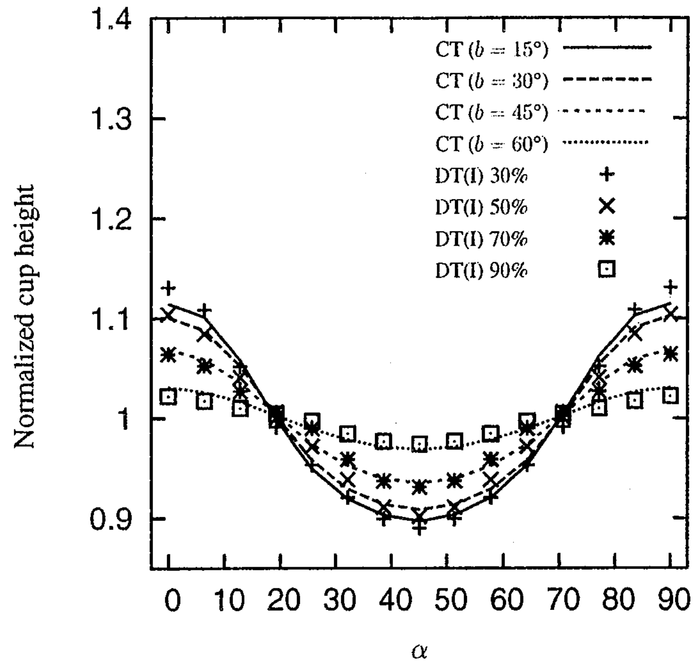

Figure 2: Earing profile for a cube texture corresponding to different isotropic volume fractions in the DT(I) model and different half-widths in the CT model

versatile.

3 Macroscopic yield criteria

The earliest anisotropic yield criterion is probably due to v. Mises [25]. The proposed quadratic form considers the anisotropy of the material by a fourth-order structure tensor. Hill [14] investigates this yield criterion for the case of orthorhombic sample symmetry. An advantage of this orthorhombic form is that all material parameters can be determined by uniaxial tests. Because of its simple form, the Hill criterion is frequently used in forming simulations.

Such an elementary approach to the description of mechanical anisotropy, however, has inherent shortcomings. For example, specific experimentally observed earing profiles cannot be reproduced by the quadratic yield function, see e.g. [23]. Furthermore, the Hill criterion cannot predict the anomalous behavior which is observed in some metal alloys [1]. In contrast to the Hill criterion, the criteria of [15, 17] allow for a description of such effects. On the other hand these criteria are limited to the case that the orthotropy axes are coincident with the principal directions of the stress. The criteria proposed by [16] and [4, 3, 2] can be generally applied. They allow for a better approximation of the anisotropic material behavior than the models of [25] and [14]. The drawback of these advanced models is the large number of parameters which is necessary to describe the anisotropy more properly.

The quadratic yield criterion of v. Mises and Hill is purely phenomenological. The parameters of the criterion can be identified by mechanical tests. By contrast the quadratic form introduced by [5] is micromechanically motivated, since it is based on a texture coefficient of fourth order [6, 7]. In the sequel the yield criterion of [14] and the criterion of [5] are compared.

The quadratic yield criterion of [25] can be brought into the form

$$\boldsymbol{\sigma} \cdot \mathbb{H}[\boldsymbol{\sigma}] - 1 = 0. \tag{1}$$

Assuming $\mathbb{H}$ as a fourth-order tensor with major symmetry and left and right minor symmetry, this tensor contains 21 independent constants describing the anisotropy of the material. In the special case of orthorhombic sample symmetry this tensor has 9 independent constants with respect to the principal anisotropy axes. If pressure-independence of the yield criterion is furthermore required, $\mathbb{H}$ is traceless in the first pair of indices. Thus, the number of independent constants for orthorhombic symmetry is reduced to 6. The result with repect to the orthotropy axes which coincide with the coordinate axes $\boldsymbol{e}_i$, is the following yield criterion [14]

$$F(\sigma_{22}-\sigma_{33})^2 + G(\sigma_{33}-\sigma_{11})^2 + H(\sigma_{11}-\sigma_{22})^2 + 2L\sigma_{23}^2 + 2M\sigma_{13}^2 + 2N\sigma_{12}^2 = 1. \tag{2}$$

Under the assumption of plane stress ($\boldsymbol{e}_1$-$\boldsymbol{e}_2$ plane), i.e., $\sigma_{33} = \sigma_{23} = \sigma_{13} = 0$, the parameters L and M do not enter the criterion (see (2)). In this case the Hill criterion only depends on four independent constants.

Following Böhlke [5], the anisotropy tensor $\mathbb{H}$ in equation (1) can be represented by

$$\mathbb{H} = \frac{3}{2\sigma_F^2}\left(\mathbb{P}_2^I + \eta\mathbb{V}'\right). \tag{3}$$

The tensor $\mathbb{P}_2^I$ is a projector of fourth order, which maps a second order tensor into its deviatoric part. $\mathbb{V}'$ characterizes the induced anisotropy. It is the fourth-order texture coefficient appearing in a tensorial Fourier expansion of the codf [6, 7]. Therefore, this tensor contains information of the crystallographic texture of the material. The influence of $\mathbb{V}'$ on the yield criterion (1) is controlled by the parameter η. When choosing $\eta = 0$, an isotropic material behavior is obtained. In comparison to the Hill tensor, $\mathbb{V}'$ is not only traceless in the first pair of indices but also completely symmetric. As a result, the number of unknowns for the tensor $\mathbb{V}'$ for triclinic symmetry can be reduced from 21 to 9. In the orthorhombic symmetry case this number can be further reduced to three.

As mentioned before, the Hill yield criterion requires the four parameters F, G, H and N when assuming plane stress conditions. If the tensor $\mathbb{H}$ from equation (3) is used and orthorhombic symmetry and plane stress conditions are assumed, then five parameters must be determined: η, V'_{1122}, V'_{1133}, V'_{2233}, σ_F. Note that only four of these parameters can be independent.

Thus, in the case of orthorhombic sample symmetry and plane stress conditions, the Hill yield criterion and the texture based yield criterion of [5] are equivalent. The correlations between the parameters of both models are given by

$$\begin{aligned} F &= \frac{1-3\eta V'_{2233}}{2\sigma_F^2} & G &= \frac{1-3\eta V'_{1133}}{2\sigma_F^2} \\ H &= \frac{1-3\eta V'_{1122}}{2\sigma_F^2} & N &= \frac{3}{2}\frac{1+2\eta V'_{1122}}{\sigma_F^2}. \end{aligned} \tag{4}$$

The coefficients V'_{1133}, V'_{2233}, and V'_{1122} can be determined by texture measurements. Furthermore, the parameters η and σ_F must be specified by, e.g., measuring the yield stress or the R value in the rolling direction

$$\eta = \frac{1}{3}\frac{R_0 - 1}{R_0 V'_{1133} - V'_{1122}} \qquad \sigma_F^2 = \frac{\sigma_0^2}{2}\frac{(R_0+1)(V'_{1133} - V'_{1122})}{R_0 V'_{1133} - V'_{1122}}. \tag{5}$$

Consequently, the identification of Hill's tensor under plane stress conditions requires the texture information and the measurement of two mechanical quantities.

4 Summary

In Chapter 2 different Taylor type models have been introduced. The models have been applied to the deep drawing process of aluminum. It has been shown that the models allow for a reduction of the anisotropy that is predicted by the standard Taylor model.

In Chapter 3 different forms for the quadratic yield criteria have been compared. In general, such yield criteria, e.g., the Hill criterion [14], are not based on microstructural informations. The anisotropy parameters are identified by mechanical tests. The yield criterion proposed by [5] is based on a fourth-order texture coefficient. This texture coefficient can be determined by texture measurements.

Under the assumption of plane stress conditions, the Hill criterion can also be identified by the texture of the material. This identification additionally requires the measurement of two mechanical quantities like, e.g., R values or yield stresses.

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