Archive of Applied Mechanics 72 (2002) 293–299 © Springer-Verlag 2002
DOI 10.1007/s00419-002-0207-z

An inelastic material model for filled polytetrafluorethylen

T. Kletschkowski, U. Schomburg, A. Bertram

Summary This paper presents a viscoplastic model for PTFE designed to simulate numerically PTFE shaft seals. A rate-independent elastoplastic model with an endochronic flow rule is coupled in series with a rate-dependent Kelvin model, which has a highly nonlinear damper. In contrast to previous models for PTFE, this unified approach is suitable for numerical simulation of the loading and the stress relaxation behaviour at ambient temperature.

Keywords Viscoplasticity, Endochronic material behaviour, Rate-dependence, Relaxation

1
Introduction

The description of inelastic material behaviour is very important for many technical applications. For example, PTFE (Polytetrafluorethylen) compounds are increasingly used for rotary shaft seals in automobiles. This is due to their tribological characteristics, chemical inertness and thermal stability. In contrast to classical rotary shaft seals, which are made of elastomeric materials, here the engineer has to take into account the inelastic material behaviour of the PTFE materials in the design process. In order to reduce expensive long tests in laboratories, inelastic material models are needed for numerical simulations.

The way of modeling the characteristic inelastic phenomena for a typical PTFE compound (90% PTFE, 5% MoS_2, 5% short cylindrical glass fibers) is illustrated in this paper. In order to develop a material model for the numerical simulation of PTFE shaft seals mechanical experiments were performed, studying material behaviour in uniaxial relaxation, uniaxial tension at different strain rates and retardation tests after unloading. The tension and relaxation tests were performed in mechanical straining of the PTFE seal during the assembling and the service life on the shaft of the engine. Straining of the seal during the assembly process produces pressure between the seal and the shaft that is required to prevent leakage. This pressure is caused by the circumferential tension stress in the sealing element, [4]. The inelastic material behaviour of the analyzed compound is discussed in [2] and illustrated by Figs. 1 and 4. Figure 1 shows that tension test specimens show after unloading the effects of strain recovery and plastic deformation. We performed in our tests a temperature cycle after unloading to accelerate the process of viscous strain recovery.

It has been observed that:

1. The relaxation curves, showing the time-dependent decrease of the measured stress, are not congruent when using a normalization based on the respective maximum value of the performed strain level.

Received 30 October 2001; accepted for publication 21 January 2002

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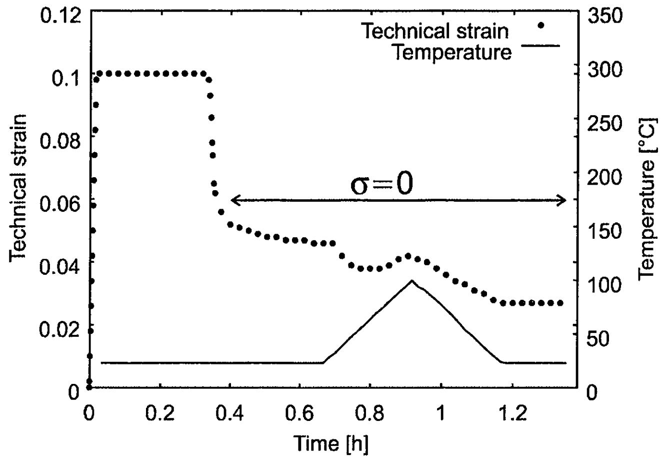

Fig. 1. Tension tests and stress relaxation

2. The stress response in the uniaxial tension tests shows a nonlinear stress-strain behaviour depending on the strain rate.
3. Mechanical straining of PTFE at ambient temperature results in a permanent inelastic deformation and strain recovery after unloading.

These results characterize the 10% compound as a viscoplastic material with nonlinear viscoelastic properties.

2
Phenomenological description of inelastic material behavior

The description of the viscoplastic phenomena observed during the tests can be embedded into a generalized concept of modeling inelastic material behaviour. Therefore we assume that

1. deformations are moderate, $\varepsilon = 10\%$;
2. we have isothermal conditions;
3. the inelastic deformation has no effect on the elastic properties.

With respect to these requirements we write the total strain tensor $\mathbf{E}$ as the sum of the elastic strain tensor $\mathbf{E}_{el}$ and the inelastic strain tensor $\mathbf{E}_{in}$ which symbolizes all parts of possible inelastic deformations.

$$\mathbf{E} = \mathbf{E}_{el} + \mathbf{E}_{in} \ . \tag{1}$$

If $\mathbf{E}_{in}$, which will be used as an internal variable, has no effect on the elastic properties of the material, we are able to formulate the following set of constitutive equations.

$$\mathbf{S} = \mathbf{S}(\mathbf{E}, \mathbf{E}_{in}) \ , \tag{2}$$

$$\psi = \psi(\mathbf{E}, \mathbf{E}_{in}) = \varphi(\mathbf{E}_{el}) \ , \tag{3}$$

$$\dot{\mathbf{E}}_{in} = \dot{\mathbf{E}}_{in}(\mathbf{E}, \dot{\mathbf{E}}, \mathbf{E}_{in}) \ . \tag{4}$$

Equation (2) describes the elastic stress response of the material, where $\mathbf{S}$ is the stress tensor. Equation (3) shows that the free energy φ depends only on the elastic part of the deformation. The rate equation for the inelastic deformation $\dot{\mathbf{E}}_{in}$ can describe either a rate-independent material behaviour or, if $\dot{\mathbf{E}}$ vanishes at the right-hand side of Eq. (4), a rate-dependent material behaviour. This set of general equations needs to be reduced to obtain specific forms of the constitutive equations. Therefore, according to our requirements, it is sufficient to analyze the second law of thermodynamics represented by

$$\delta = \mathbf{S} \cdot \dot{\mathbf{E}} - \dot{\varphi} \geq 0 \ , \tag{5}$$

that describes the rate of dissipation during an inelastic deformation process. Inserting the time derivative of φ into Eq. (5) and using Eq. (1), we obtain the equation for the state of stress and the dissipation inequality

$$\mathbf{S} = \frac{\partial \varphi}{\partial \mathbf{E}_{el}} \ , \tag{6}$$

$$\mathbf{S} \cdot \dot{\mathbf{E}}_{in} \geq 0 \ . \tag{7}$$

The constitutive model is thermodynamically consistent, if the constitutive equation (6) and the inequality of dissipation (7) hold. If the free energy is represented by

$$\varphi = \frac{E}{2(1+\nu)} \left[\mathbf{E}_{el} \cdot \mathbf{E}_{el} + \frac{\nu}{1-2\nu} \mathrm{tr}^2(\mathbf{E}_{el}) \right] \ , \tag{8}$$

we obtain the generalization of Hooke's law for the state of stress, with the material constants E (Young's modulus) and ν (Poisson's ratio):

$$\mathbf{S} = \frac{E}{(1+\nu)} \left[\mathbf{E}_{el} + \frac{\nu}{1-2\nu} \mathrm{tr}(\mathbf{E}_{el}) \mathbf{I} \right] \ . \tag{9}$$

If we assume that all inelastic deformations are isochoric, we are able to establish a generalized flow rule in the following way. First, we define an equivalent stress

$$\sigma_V := \sqrt{\frac{3}{2} \mathrm{tr}(\mathbf{S}^{*dev} \mathbf{S}^{*dev})} \ . \tag{10}$$

Here $\mathbf{S}^*$ stands for the effective stress tensor that influences the inelastic flow. To obtain an associated flow rule we use

$$\phi := \frac{1}{2} \sigma_V^2 \ , \tag{11}$$

as a potential for the rate of the inelastic deformation

$$\dot{\mathbf{E}}_{in} = \lambda \frac{\partial \phi}{\partial \mathbf{S}} = \frac{3}{2} \lambda \mathbf{S}^{*dev} \ . \tag{12}$$

Equation (12) shows that the direction of the inelastic flow is determined by the direction of the deviatoric part of the effective stress tensor. The proportional factor λ can be calculated using the following equation

$$\mathbf{S} \cdot \dot{\mathbf{E}}_{in} = \sigma_V \dot{\varepsilon}_{inV} \Rightarrow \ \lambda = \frac{\dot{\varepsilon}_{inV}}{\sigma_V} \ , \tag{13}$$

$$\dot{\varepsilon}_{inV} := \sqrt{\frac{2}{3} \mathrm{tr}(\dot{\mathbf{E}}_{in} \dot{\mathbf{E}}_{in})} \tag{14}$$

The associated flow rule,

$$\dot{\mathbf{E}}_{in} = \frac{3}{2} \frac{\dot{\varepsilon}_{inV}}{\sigma_V} \mathbf{S}^{*dev} \tag{15}$$

for the rate of the inelastic deformation depends on the rate of the equivalent inelastic flow, see (14), the equivalent stress, and the deviatoric part of the effective stress tensor. The quantity $\mathbf{S}^{*dev}/\sigma_V$. defines the direction of the inelastic flow, and $3/2\dot{\varepsilon}_{inV}$ defines its amplitude.

To formulate a specific flow rule due to special inelastic phenomena we need a constitutive equation between the equivalent inelastic strain rate and the equivalent stress. Inelastic material behaviour can be modeled by

$$\sigma_V \mapsto \dot{\varepsilon}_{inV} = \frac{1}{g(\sigma_V, \varepsilon_{inV}, \dot{\varepsilon}_V, \theta - \theta_0)} f(\sigma_V) \ , \tag{16}$$

$$\dot{\varepsilon}_V := \sqrt{\frac{2}{3}\mathrm{tr}(\dot{\mathbf{E}}\dot{\mathbf{E}})} \ , \tag{17}$$

where $\dot{\varepsilon}_V$ is the equivalent total strain rate. If the scalar-valued functions g and f are positive, the dissipation inequality (7) is fulfilled. The set of reduced constitutive equations for the description of inelastic material behaviour for isothermal processes is summarized in Table 1.

Now we specify the general constitutive equation (16) for the phenomena revealed in the uniaxial tests at reference temperature $\theta_0 = 23\,°\mathrm{C}$ for the PTFE compound. The test results show that the inelastic deformation is a sum of viscoelastic and plastic deformations

$$\mathbf{E}_{\mathrm{in}} = \mathbf{E}_v + \mathbf{E}_p \ . \tag{18}$$

We model therefore the viscoplastic material behaviour of the compound by a generalization of the rheological model shown in Fig. 2. It consists of an endochronic elastoplastic element and a Kelvin element. The theory of endochronic material behaviour was discussed in detail in [1], [3] and [7]. An illustration will be given later.

For the generalization of the rheological model to three dimensions we choose the identifications listed in Table 2. The viscous backstress $\mathbf{S}_\mathrm{B}$ in our approach is proportional to the viscous deformation.

The scalar-valued rate equations for the equivalent inelastic rates can be directly adopted from the rheological model. For the equivalent viscous flow rate we choose the following differential equation:

$$\sigma_V \mapsto \dot{\varepsilon}_{vV} = \frac{1}{\eta_0}\frac{1}{1 + (\kappa_0 + \kappa_D \tanh[B\dot{\varepsilon}_V])\varepsilon_{vV}} \exp[Z\sigma_V]\sigma_V \ , \tag{19}$$

where Z, κ_0 and κ_D are model parameters. The rate equation (19) is highly nonlinear and allows to include rate effects in the viscosity. For high positive values of the parameter B, the viscosity depends on the equivalent total strain rate. For low positive values of B, the viscosity differs only slightly if the deformation is constant ($\dot{\varepsilon}_V = 0$) or not ($\dot{\varepsilon}_V \neq 0$). An example of the use of rate-dependent viscosity functions to model inelastic material behaviour can be found in [6]. For the equivalent plastic flow rate we choose the following rate equation:

$$\sigma_V \mapsto \dot{\varepsilon}_{pV} = \frac{1}{Y}\sigma_V\dot{\varepsilon}_V \ , \tag{20}$$

with $Y = \mathrm{const}$. In the one-dimensional case, the combination of Hooke's law

$$\sigma = E(\varepsilon - \varepsilon_p) \Rightarrow \dot{\sigma} = E(\dot{\varepsilon} - \dot{\varepsilon}_p) \ , \tag{21}$$

and an uniaxial endochronic elastoplastic flow rule

$$\sigma \mapsto \dot{\varepsilon}_p = \frac{1}{Y}\sigma|\dot{\varepsilon}| \tag{22}$$

with $Y = \mathrm{const}$, leads to a differential equation of the Armstrong–Frederick type. This is a saturation-type evolution equation.

Table 1. General constitutive equations

Constitutive decomposition	$\mathbf{E} = \mathbf{E}_{\mathrm{el}} + \mathbf{E}_{\mathrm{in}}$
Elastic law	$\mathbf{S} = \frac{\partial\varphi}{\partial\mathbf{E}_{\mathrm{el}}}$
Associated flow rule	$\dot{\mathbf{E}}_{\mathrm{in}} = \frac{3}{2}\frac{\dot{\varepsilon}_{\mathrm{in}V}}{\sigma_V}\mathbf{S}^{*\mathrm{dev}}$

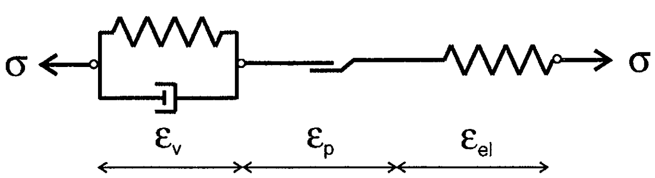

Fig. 2. Rheological model

Table 2. Generalization of the rheological model

	Kelvin model	Endochronic model
$\dot{\mathbf{E}}_{in} \equiv$	$\dot{\mathbf{E}}_v$	$\dot{\mathbf{E}}_p$
$\dot{\varepsilon}_{inV} \equiv$	$\dot{\varepsilon}_{vV}$	$\dot{\varepsilon}_{pV}$
$\mathbf{S}^* \equiv$	$\mathbf{S} - \mathbf{S}_B$	$\mathbf{S}$
$\sigma_V \equiv$	$\sigma_V(\mathbf{S}^{dev} - \mathbf{S}_B)$	$\sigma_V(\mathbf{S}^{dev})$

$$\dot{\sigma} = E\left(\dot{\varepsilon} - \frac{1}{Y}\sigma|\dot{\varepsilon}|\right) . \tag{23}$$

For an isothermal tension test with a constant strain rate, Eq. (23) can be integrated

$$\sigma = Y\left(1 - \exp\left[-\frac{E}{Y}\varepsilon\right]\right) , \tag{24}$$

with σ as the uniaxial stress and ε the total uniaxial strain.

The stress-strain characteristic of purely elastoplastic endochronic material behaviour in one dimension is illustrated by Fig. 3. This shows that Y defines the maximum stress that can be reached during the deformation process.

The set of constitutive equations for the description of the viscoplastic material behaviour of the analyzed PTFE compound at ambient temperature is summarized in Table 3.

3
Identification and numerical results

For the identification of the model parameters we used a tension test (constant strain rate of $\dot{\varepsilon} = 10\%/\text{min}$ and $\varepsilon_{max} = 8\%$ at ambient temperature), the results of the succeeding stress relaxation and the size of the plastic deformation after unloading and strain recovery. In the identification process, all model parameters were first identified for the one-dimensional rheological model and then adjusted to its three-dimensional generalization. The elastic constants were directly determined from the loading curve of the tension test. For the identification of the maximum stress of the elastoplastic endochronic element we defined the area under the loading curve.

$$\Xi := \int_{\varepsilon=\varepsilon_0}^{\varepsilon=\varepsilon_1} \sigma \, d\varepsilon . \tag{25}$$

The size of the plastic deformation after unloading and strain recovery is calculated as $\varepsilon_p = 2.4\%$. Using the rate equation of the endochronic flow rule, the maximum stress Y can be calculated from:

$$Y = \frac{\Xi}{\varepsilon_p} . \tag{26}$$

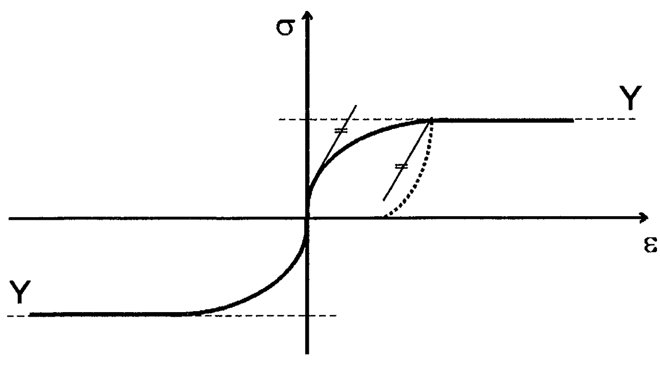

Fig. 3. $\sigma - \varepsilon$ curve of an endochronic elastoplastic element

Table 3. Constitutive equations

Elastic law	$\mathbf{S} = \frac{E}{(1+\nu)}\left[\mathbf{E}_{\text{el}} + \frac{\nu}{1-2\nu}\text{tr}(\mathbf{E}_{\text{el}})\mathbf{I}\right]$
Endochronic flow rule	$\dot{\mathbf{E}}_p = \frac{3}{2}\frac{1}{Y}\mathbf{S}^{\text{dev}}\dot{\varepsilon}_V$
Viscous flow rule	$\dot{\mathbf{E}}_v = \frac{3}{2}\frac{1}{\eta}\left(\mathbf{S}^{\text{dev}} - \mathbf{S}_{\text{B}}\right)$
	$\eta = \eta_0 \exp[-Z\sigma_V](1 + \kappa\varepsilon_{vV})$
	$\kappa = \kappa_0 + \kappa_D \tanh[B\dot{\varepsilon}_V]$
Backstress	$\mathbf{S}_{\text{B}} = C_{\text{B}}\mathbf{E}_v$

Table 4. Parameters of the model at θ_0

$E = 690.0$ N/mm^2	$\eta_0 = 240,5 \cdot 10^6$ Ns/mm^2
$C_{\text{B}} = 16.65$ N/mm^2	$\kappa_0 = 1,193 \cdot 10^6$
$Y = 26.1$ N/mm^2	$\kappa_D = 7,528 \cdot 10^6$
$B = 1 \cdot 10^6$	$Z = 2.35$ mm^2/N

To include rate effects for the viscosity, a high positive value for B was selected. The rest of the model parameters were identified using the Levenberg-Marquardt algorithm, [5]. First, η_0, κ_0 and Z were obtained by using this algorithm, to minimize the sum of least squares of the differences between the measurement and numerical calculation of the stress relaxation. Then, κ_D was identified by using this algorithm to minimize the sum of least squares of the differences between the measurement and numerical calculation of the tension test. Because of the endochronic flow rule, we do not need to evaluate a flow limit and a consistency condition during the numerical computations. This accelerates in general, the identification and the simulations. The set of parameters for the description of the viscoplastic material behaviour of the analyzed PTFE compound at ambient temperature is summarized in Table 4. For all simulations, a Poisson's ratio of $\nu = 0.46$ was chosen, [4].

The results of numerical simulations of tension tests at different strain rates ($\dot{\varepsilon} = 1\%/\text{min}, 10\%/\text{min}, 100\%/\text{min}$) and relaxation tests after tension tests with a constant strain rate of $\dot{\varepsilon} = 10\%/\text{min}$ but at different strain levels ($\varepsilon = 3\%, 5\%, 8\%$) calculated by the three-dimensional model are shown in Fig. 4. The comparison of the results of the numerical simulations with the experimental data show that:

1. The approximation of the loading and relaxation curve used for the identification is satisfactory.
2. The quality of the simulation of the loading and relaxation curves at different strain levels using the same strain rate is also satisfactory.
3. The model allows to predict the strain-rate dependence of the stress response caused by loading processes at different strain rates.

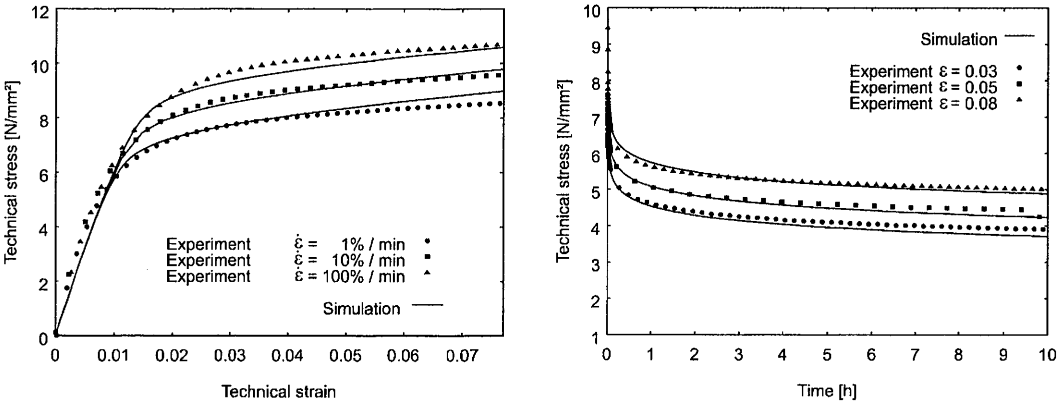

Fig. 4. Numerical simulations

4
Conclusions

The paper presents a viscoplastic model for the description of the inelastic material behaviour of a typical PTFE compound at isothermal deformation processes. The model is included into a general concept based on the additive decomposition of the total strain tensor into elastic and inelastic parts. The three-dimensional constitutive equations were obtained from a generalization of a rheological model that consists of an endochronic elastoplastic element coupled in series with a nonlinear Kelvin element. The numerical simulation shows that the model is suitable to compute the inelastic phenomena observed in the experiment.

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Introduction

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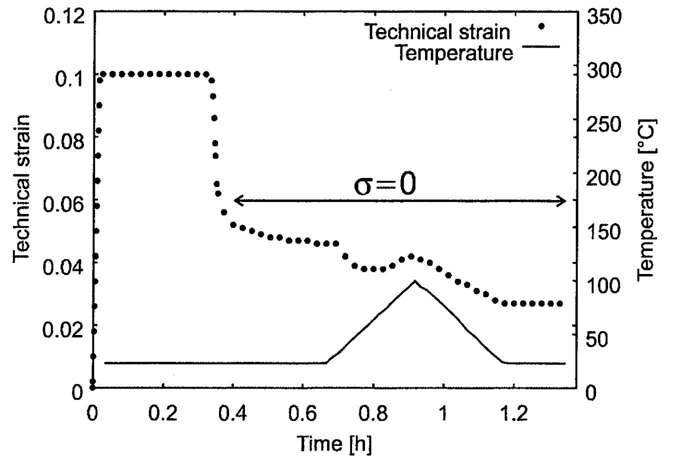

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The scalar-valued rate equations for the equivalent inelastic rates can be directly adopted from the rheological model. For the equivalent viscous flow rate we choose the following differential equation:

$$\sigma_V \mapsto \dot{\varepsilon}_{vV} = \frac{1}{\eta_0}\frac{1}{1 + (\kappa_0 + \kappa_D \tanh[B\dot{\varepsilon}_V])\varepsilon_{vV}}\exp[Z\sigma_V]\sigma_V \ , \tag{19}$$

where Z, κ_0 and κ_D are model parameters. The rate equation (19) is highly nonlinear and allows to include rate effects in the viscosity. For high positive values of the parameter B, the viscosity depends on the equivalent total strain rate. For low positive values of B, the viscosity differs only slightly if the deformation is constant ($\dot{\varepsilon}_V = 0$) or not ($\dot{\varepsilon}_V \neq 0$). An example of the use of rate-dependent viscosity functions to model inelastic material behaviour can be found in [6]. For the equivalent plastic flow rate we choose the following rate equation:

$$\sigma_V \mapsto \dot{\varepsilon}_{pV} = \frac{1}{Y}\sigma_V\dot{\varepsilon}_V \ , \tag{20}$$

with $Y = \mathrm{const}$. In the one-dimensional case, the combination of Hooke's law

$$\sigma = E(\varepsilon - \varepsilon_p) \Rightarrow \dot{\sigma} = E(\dot{\varepsilon} - \dot{\varepsilon}_p) \ , \tag{21}$$

and an uniaxial endochronic elastoplastic flow rule

$$\sigma \mapsto \dot{\varepsilon}_p = \frac{1}{Y}\sigma|\dot{\varepsilon}| \tag{22}$$

with $Y = \mathrm{const}$, leads to a differential equation of the Armstrong–Frederick type. This is a saturation-type evolution equation.

Table 1. General constitutive equations

Constitutive decomposition	$\mathbf{E} = \mathbf{E}_{\mathrm{el}} + \mathbf{E}_{\mathrm{in}}$
Elastic law	$\mathbf{S} = \frac{\partial\varphi}{\partial\mathbf{E}_{\mathrm{el}}}$
Associated flow rule	$\dot{\mathbf{E}}_{\mathrm{in}} = \frac{3}{2}\frac{\dot{\varepsilon}_{\mathrm{in}V}}{\sigma_V}\mathbf{S}^{*\mathrm{dev}}$

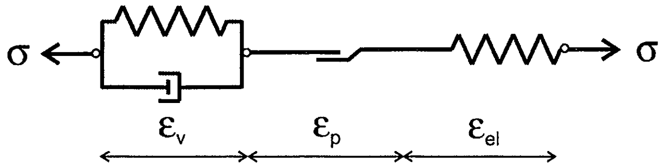

Fig. 2. Rheological model

Table 2. Generalization of the rheological model

	Kelvin model	Endochronic model
$\dot{\mathbf{E}}_{in} \equiv$	$\dot{\mathbf{E}}_v$	$\dot{\mathbf{E}}_p$
$\dot{\varepsilon}_{inV} \equiv$	$\dot{\varepsilon}_{vV}$	$\dot{\varepsilon}_{pV}$
$\mathbf{S}^* \equiv$	$\mathbf{S} - \mathbf{S}_B$	$\mathbf{S}$
$\sigma_V \equiv$	$\sigma_V(\mathbf{S}^{dev} - \mathbf{S}_B)$	$\sigma_V(\mathbf{S}^{dev})$

$$\dot{\sigma} = E\left(\dot{\varepsilon} - \frac{1}{Y}\sigma|\dot{\varepsilon}|\right) . \tag{23}$$

For an isothermal tension test with a constant strain rate, Eq. (23) can be integrated

$$\sigma = Y\left(1 - \exp\left[-\frac{E}{Y}\varepsilon\right]\right) , \tag{24}$$

with σ as the uniaxial stress and ε the total uniaxial strain.

The stress-strain characteristic of purely elastoplastic endochronic material behaviour in one dimension is illustrated by Fig. 3. This shows that Y defines the maximum stress that can be reached during the deformation process.

The set of constitutive equations for the description of the viscoplastic material behaviour of the analyzed PTFE compound at ambient temperature is summarized in Table 3.

3
Identification and numerical results

For the identification of the model parameters we used a tension test (constant strain rate of $\dot{\varepsilon} = 10\%/\text{min}$ and $\varepsilon_{max} = 8\%$ at ambient temperature), the results of the succeeding stress relaxation and the size of the plastic deformation after unloading and strain recovery. In the identification process, all model parameters were first identified for the one-dimensional rheological model and then adjusted to its three-dimensional generalization. The elastic constants were directly determined from the loading curve of the tension test. For the identification of the maximum stress of the elastoplastic endochronic element we defined the area under the loading curve.

$$\Xi := \int_{\varepsilon=\varepsilon_0}^{\varepsilon=\varepsilon_1} \sigma \, d\varepsilon . \tag{25}$$

The size of the plastic deformation after unloading and strain recovery is calculated as $\varepsilon_p = 2.4\%$. Using the rate equation of the endochronic flow rule, the maximum stress Y can be calculated from:

$$Y = \frac{\Xi}{\varepsilon_p} . \tag{26}$$

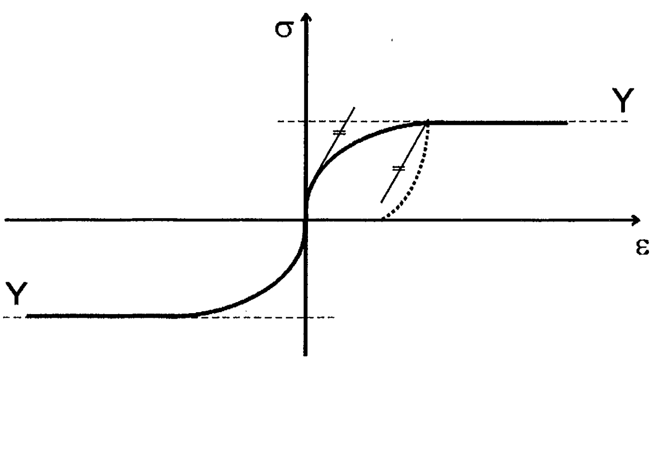

Fig. 3. $\sigma - \varepsilon$ curve of an endochronic elastoplastic element

Table 3. Constitutive equations

Elastic law	$\mathbf{S} = \frac{E}{(1+\nu)}\left[\mathbf{E}_{el} + \frac{\nu}{1-2\nu}\mathrm{tr}(\mathbf{E}_{el})\mathbf{I}\right]$
Endochronic flow rule	$\dot{\mathbf{E}}_p = \frac{3}{2}\frac{1}{Y}\mathbf{S}^{dev}\dot{\varepsilon}_V$
Viscous flow rule	$\dot{\mathbf{E}}_\nu = \frac{3}{2}\frac{1}{\eta}\left(\mathbf{S}^{dev} - \mathbf{S}_B\right)$
	$\eta = \eta_0 \exp[-Z\sigma_V](1 + \kappa\varepsilon_{\nu V})$
	$\kappa = \kappa_0 + \kappa_D \tanh[B\dot{\varepsilon}_V]$
Backstress	$\mathbf{S}_B = C_B \mathbf{E}_\nu$

Table 4. Parameters of the model at θ_0

$E = 690.0$ N/mm^2	$\eta_0 = 240{,}5 \cdot 10^6$ Ns/mm^2
$C_B = 16.65$ N/mm^2	$\kappa_0 = 1{,}193 \cdot 10^6$
$Y = 26.1$ N/mm^2	$\kappa_D = 7{,}528 \cdot 10^6$
$B = 1 \cdot 10^6$	$Z = 2.35$ mm^2/N

To include rate effects for the viscosity, a high positive value for B was selected. The rest of the model parameters were identified using the Levenberg-Marquardt algorithm, [5]. First, η_0, κ_0 and Z were obtained by using this algorithm, to minimize the sum of least squares of the differences between the measurement and numerical calculation of the stress relaxation. Then, κ_D was identified by using this algorithm to minimize the sum of least squares of the differences between the measurement and numerical calculation of the tension test. Because of the endochronic flow rule, we do not need to evaluate a flow limit and a consistency condition during the numerical computations. This accelerates in general, the identification and the simulations. The set of parameters for the description of the viscoplastic material behaviour of the analyzed PTFE compound at ambient temperature is summarized in Table 4. For all simulations, a Poisson's ratio of $\nu = 0.46$ was chosen, [4].

The results of numerical simulations of tension tests at different strain rates ($\dot{\varepsilon} = 1\%/\mathrm{min}, 10\%/\mathrm{min}, 100\%/\mathrm{min}$) and relaxation tests after tension tests with a constant strain rate of $\dot{\varepsilon} = 10\%/\mathrm{min}$ but at different strain levels ($\varepsilon = 3\%, 5\%, 8\%$) calculated by the three-dimensional model are shown in Fig. 4. The comparison of the results of the numerical simulations with the experimental data show that:

1. The approximation of the loading and relaxation curve used for the identification is satisfactory.
2. The quality of the simulation of the loading and relaxation curves at different strain levels using the same strain rate is also satisfactory.
3. The model allows to predict the strain-rate dependence of the stress response caused by loading processes at different strain rates.

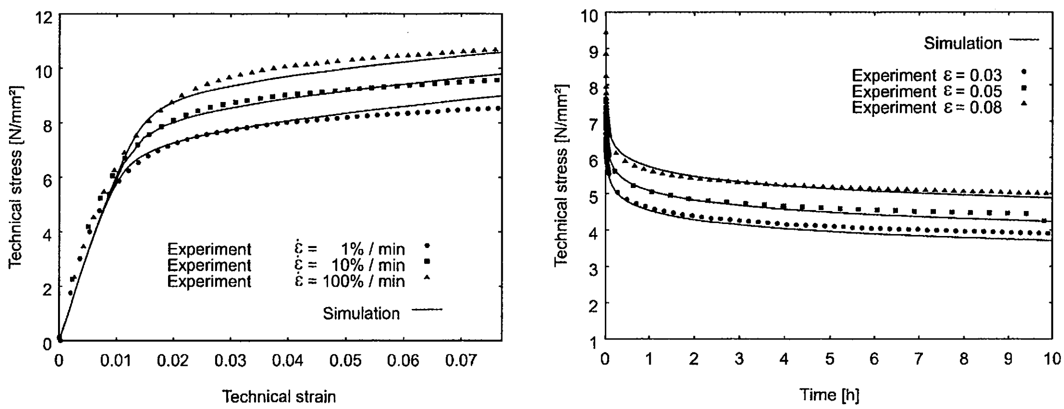

Fig. 4. Numerical simulations

4
Conclusions

The paper presents a viscoplastic model for the description of the inelastic material behaviour of a typical PTFE compound at isothermal deformation processes. The model is included into a general concept based on the additive decomposition of the total strain tensor into elastic and inelastic parts. The three-dimensional constitutive equations were obtained from a generalization of a rheological model that consists of an endochronic elastoplastic element coupled in series with a nonlinear Kelvin element. The numerical simulation shows that the model is suitable to compute the inelastic phenomena observed in the experiment.

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