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Identification of Elastic Constants of Directionally Solidified Superalloys based on Resonance Measurements and FE-Analysis

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ABSTRACT

As an inverse problem, the identification of the five independent elastic constants of directionally solidified (DS) superalloys at elevated temperatures based on resonance measurements is presented in this paper. The identification strategy consists in adjusting the elastic constants by minimizing the differences between calculated and measured natural frequencies of several specimens. It includes a non-linear optimizer, an FE-programm for solving eigenvalue equations and a frequency assignment algorithm. The results for the Nickel-based superalloy IN 738 LC DS demonstrate the efficiency and the economy of the method.

1. Introduction

By controlled solidification technique, the single crystal and directionally solidified superalloys have been developed and are used more and more as advanced turbine blade materials for improving the gas turbine engine efficiency. The main advantages claimed for these materials are improvements of the thermal fatigue resistance, of the ductility and the creep rupture lives. For design of engines with these materials the knowledge of the elastic constants is a fundamental requirement, especially for the analyses of structural vibration, strength, fracture, or stability. The authors have proposed a method[1] for identifying the elastic constants of single crystal superalloys based on resonance measurements and FE-analysis. It is a systematic extension of the Förster resonance method[2] for anisotropic materials.

DS superalloys exhibit transverse isotropy with five independent elastic constants. The natural frequencies of a specimen consisting of a DS superalloy are determined by its inertia and stiffness properties. The stiffness property is completely described by the orientation and the five elastic constants. The orientation can be represented by the polar angle between the columnar grain growth direction and the longitudinal axis of the specimen. By a suitable approximate method, e.g., FEM, it is possible to calculate the natural frequencies of the specimen, if the five constants, the orientation, the density, and the geometry of the specimen are known.

In our case, however, it deals with the inverse problem: determination of the five elastic constants using measured natural frequencies. This paper presents an extension of the method[1] for carrying out the determination by FE-analysis. It includes a non-linear optimizer, an FE-programm for solving eigenvalue equations taking into account exactly the transversely isotropic constitutive law and the boundary conditions of the free-vibrating specimen, and a frequency assignment algorithm. The results for the superalloy IN 738 LC DS demonstrate the efficiency and the economy of the method.

2. Transverse Isotropy

DS superalloys of cubic crystal structure in general show a strong [001]-fibre texture which can be described by a hexagonal symmetry. This exhibits a transverse isotropy. Relative to the Cartesian coordinate system (i, j, k), where the k-axis is parallel to the solidification direction, the strain and stress vectors in the Hooke's law are related by following symmetrical compliance matrix[3]:

$$S = \begin{vmatrix} S_{11} & S_{12} & S_{13} & 0 & 0 & 0 \\ & S_{11} & S_{13} & 0 & 0 & 0 \\ & & S_{33} & 0 & 0 & 0 \\ & & & S_{44} & 0 & 0 \\ & & & & S_{44} & 0 \\ & & & & & 2(S_{11} - S_{12}) \end{vmatrix} \qquad (1)$$

where S_{11}, S_{12}, S_{13}, S_{33}, and S_{44} are the five independent elastic constants in compliance. The five constants and the structure of the matrix S describe the transversely isotropic (hexagonal anisotropic) behaviour. The five constants can also be represented by the five stiffnesses C_{11}, C_{12}, C_{13}, C_{33}, and C_{44} or the Young's and shear moduli E_a, E_c, G_{ab}, G_{ac} and the Poisson's ratio υ_{ac}. The relationships between these three representations are shown in Appendix A. With respect to the specimen-fixed Cartesian coordinate system (x, y, z), where the z-axis lies parallel to the specimen axis, the compliance matrix, noted by S*, can be formed by the five constants in the formula (1) and the polar angle θ which defines the angle between k- and z-axes[4]. The two coordinate systems are shown in Fig. 1. For

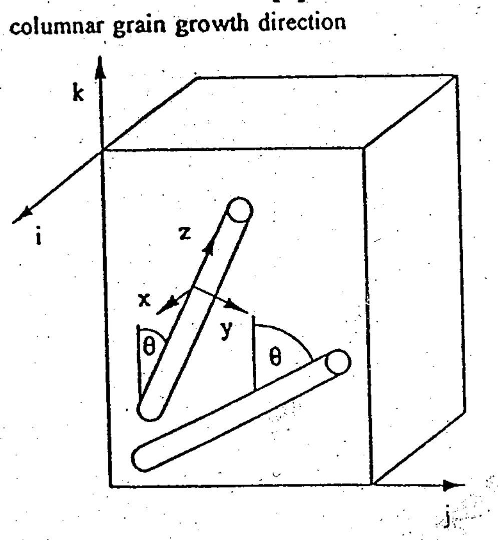

Fig. 1 - Coordinate Systems

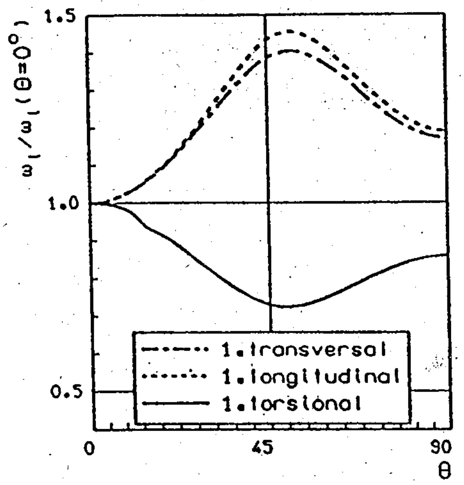

Fig. 2 - Dependence of Frequencies on Orientation

instance, the component S_{33}^* of the matrix S^* is given by

$$S_{33}^* = S_{11} \sin^4\theta + S_{33} \text{con}^4\theta + (2S_{13} + S_{44}) \sin^2\theta \text{ con}^2\theta. \qquad (2)$$

The aim of this paper is to identify the five independent elastic constants of DS superalloys by measured natural frequencies. The of the measurements of the natural frequencies are described in the references[1,4]. From these evaluated five constants the three independent constants of the single crystal with cubic crystal structure can be known quantitatively.

3. Dependence of Frequencies on Orientation and Elastic Constants

For a cylindrical specimen consisting of isotropic material which has two independent elastic constants, analytic solutions have been worked out for the longitudinal, transversal, and torsional vibration modes. Based on these solutions the two constants: the Young's modulus and the shear modulus, can be calculated directly[2]. However, this is not valid for the DS superalloys because of the hexagonal anisotropy. As a result, the two frequencies of a transversal natural mode pair of a DS superalloy cylindrical specimen are not identical, if the polar angle θ is not just equal to zero. In our case, each of the specimens is modeled with isoparametric 20-node-elements. The eigenvalue equations are given by

$$(-\omega_i^2 \mathbf{M} + \mathbf{K}).\mathbf{u}_i = 0, \qquad i = 1, 2, ..., \qquad (3)$$

where ω_i and $\mathbf{u}_i$ are the i-th natural frequency and mode, M and K are the inertia and stiffness matrices of the FE-mode, respectively. Fig. 2 shows the dependence of the first transversal, longitudinal, and torsional natural frequencies on the polar angle θ, and Fig. 3 shows the dependence of these frequencies on the constants E_a, E_c, G_{ab}, G_{ac} and υ_{ac}. From Fig. 2 and 3, following facts can be concluded:

a) These frequencies have the lowest gradient near of θ = 0°, 45°, and 90°, respectively.

b) The four constants E_a, E_c, G_{ab}, and G_{ac} can be identified by the transversal and torsional natural frequencies of two specimens whose polar angles are equal to 0° and 90°, respectively. The sensitivity of the transversal and torsional frequencies of a specimen with a polar angle of 45° on the Poisson's ratio υ_{ac} is obviously greater than that of the two above mentioned specimens. Taking this into account it can be concluded that the five constants can be identified by several lowest transversal and torsional natural frequencies of three specimens which have the polar angles 0°, 45°, and 90°, respectively.

c) A great advantage of the use of the above proposed three specimens is that the experimental errors in the polar angles have relatively small influence on the natural frequencies because the lowest gradient occurs near θ = 0°, 45°, or 90°.

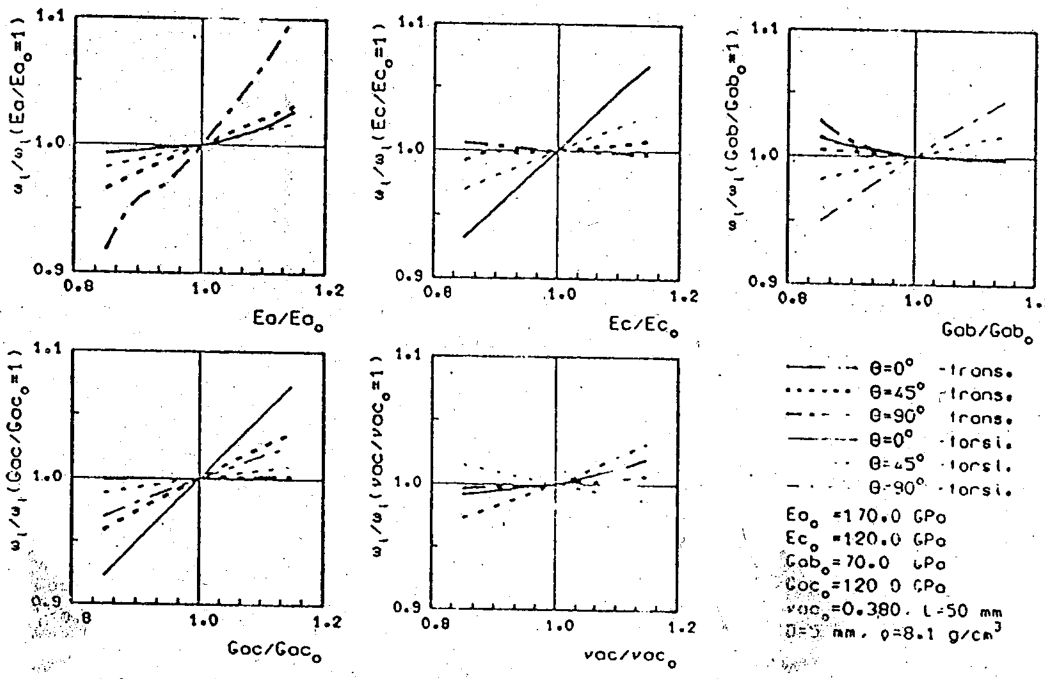

Fig. 3 - Dependence of Natural Frequencies on Elastic Constants

4. Optimization Procedure

The strategy of the identification consists in the adjustment of the five constants by an optimization procedure by minimizing the differences between the measured and calculated natural frequencies of several specimens. The object function of the optimizer is the square sum of these differences. The variables of this function are the five constants: E_a, E_c, G_{ab}, G_{ac} and υ_{ac}. The Egs.(3) are solved by the FEM code ADINA[5]. The optimizer used here is from MINPACK[6]. It uses a modified Levenberg-Marquard algorithm. The Jacobian is calculated by a forward-difference approximation. In every iteration step of the optimization procedure, the calculated frequencies must be assigned to the measured ones. This is carried out using the orthogonality of natural modes. The convergence is controlled by three parameters which measure the relative errors desired in the object function and in the approximation solutions as well as the orthogonality desired between the object function vector and the columns of the Jacobian.

5. Application

As a practical application, we consider the results for the superalloy IN 738 LC DS. Three specimens of the superalloy are used for the identification. The polar angles of the specimens are 0°, 45°, and 90°, respectively. They are determined by the Laue back-reflection x-ray technique. The starting values for the variables are rather rough estimations. The convergence was achieved satisfactorily. Tab.1 presents the identified constants at room temperature. Tab. 2 shows the comparison of the measured and calculated natural frequencies. The symbols M, C, and D in Tab.2 represent the measured frequencies, calculated frequencies, and the relative deviations of these frequencies, respectively. The algebraic average and variance of the deviations are equal to -0.00016 and 0.0083, respectively. The smallness of the two values shows the efficiency of the method and the good agreement between the calculated and measured natural frequencies of the three specimen. This allows the conclusion that the identified constants can be regarded as a good approximation for the exact ones of the superalloy IN 738 LC DS.

Tab. 1 - Identified Elastic Constants of IN 738 LC DS

S_{11} [10^{-12}/Pa]	S_{12} [10^{-12}/Pa]	S_{13} [10^{-12}/Pa]	S_{33} [10^{-12}/Pa]	S_{44} [10^{-12}/Pa]
5.920	-1.043	-3.182	8.249	8.161
E_a [GPa]	E_c [GPa]	G_{ab} [GPa]	G_{ac} [GPa]	υ_{ac}
168.9	121.2	71.81	122.5	0.3858

6. Conclusion

It is possible to identify the elastic constants of DS superalloys, if several natural frequencies of specimens under test can be precisely measured. The successful identification of the elastic constants are attributed to the accurate measurements and calculations of the natural frequencies, the suitable optimizer, and the reliable frequency assignment algorithm in every iteration step of the optimization procedure.

Tab. 2 - Comparison of calculated and measured natural frequencies

type	specimen 1 (θ=0°)			specimen 2 (θ=45°)			specimen 3 (θ=90°)		
	M [kHz]	C [kHz]	D [%]	M [kHz]	C [kHz]	D [%]	M [kHz]	C [kHz]	D [%]
tran.	7.134	7.125	0.12	7.721	7.739	0.24	35.51	35.19	0.92
tran.	7.186	7.125	0.84	7.761	7.876	1.49	36.30	36.21	0.25
tran.	19.47	19.35	0.59	20.59	20.66	0.33	83.60	83.09	0.60
tran.	19.52	19.35	0.85	20.85	21.25	1.91	87.93	87.90	0.03
tran.	37.46	37.21	0.67	38.54	38.77	0.59	141.1	139.5	1.13
tran.	37.50	37.21	0.77	39.61	40.47	2.18	150.3	151.3	0.68
tors.	50.79	51.09	0.58	32.54	32.55	0.02	673.2	670.8	0.36
tors.	102.9	102.2	0.65	65.85	65.51	0.52	133.8	134.3	0.39
tors.	154.0	153.3	0.43	97.85	98.04	0.19	202.4	201.8	0.32

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Appendix A

$E_a = 1/S_{11}$, $E_c = 1/S_{33}$, $G_{ab} = 1/[2(S_{11} - S_{12})]$, $G_{ac} = 1/S_{44}$, $\upsilon_{ac} = - S_{13}/S_{33}$.

$A = S_{11}S_{33} - S_{13}^2$, $B = S_{11}S_{13} - S_{12}S_{13}$, $D = (S_{11}^2 - S_{12}^2)A - B^2$,

$C_{11} = [S_{33}D + S_{11} (S_{12}S_{33} - S_{13}^2)^2]/AD$, $C_{12} = - (S_{12}A - S_{13}B)/D$,

$C_{13} = [B(S_{12}A - S_{13}B) - S_{13}D]/AD$, $C_{33} = S_{11}(S_{11}^2 - S_{12}^2)/D$, $C_{44} = 1/S_{44}$.