

## ANISOTROPIC CONTINUUM DAMAGE MODELLING FOR F.C.C.-SINGLE CRYSTALS AT HIGH TEMPERATURES

Albrecht Bertram<sup>1</sup>, Weidong Qi<sup>2</sup>

<sup>1</sup> Otto-von-Guericke-Universität Magdeburg, Institut für Mechanik,  
Universitätsplatz 2, D-39106 Magdeburg, Germany  
bertram@mb.uni-magdeburg.de

<sup>2</sup> Technische Universität Berlin, Institut für Mechanik,  
Straße des 17. Juni 135, D-10623 Berlin, Germany  
weidong@mech2.pi.tu-berlin.de

**Abstract-** In single crystals, the process of creep damage is anisotropic. Indeed, the damage evolution not only depends on the loading conditions, but also on the lattice orientations. And the current state of damage has an anisotropic influence on the effective stress state, so that it is represented by a tensorial damage variable. The damage model has been implemented in a three-dimensional anisotropic creep model. It is shown that the resulting material model is capable of describing the orientation dependence of the creep and damage evolution of nickel-based superalloys in the high temperature regime.

**INTRODUCTION:** It is well-known that the creep process of a metal is accompanied by the growth and nucleation of microcracks. Especially in the tertiary creep phase, this is the dominant mechanism, which causes the formation and coalescence of microcracks and finally leads to rupture. As a phenomenological approach to tertiary creep under uniaxial tensile load, Kachanov and Rabotnov introduced a scalar parameter to represent the loss of load-carrying cross-section which takes place due to damage (see Kachanov [1986]). This idea has resulted in the development of continuum damage mechanics, and has found wide application.

Single crystal superalloys are of increasing importance, especially in the turbine industry. Viscoplastic damage approaches and lifetime prediction under multidimensional loading conditions are of great interest. For the description of the primary and secondary creep phases of cubic single crystals, a phenomenological anisotropic creep-model has been suggested by Bertram, Olschewski [1996]. It is based on the 4-parameter Burgers-model which nicely reproduces the primary and secondary creep behavior. The nonlinearity between creep strains and applied stresses is incorporated into this model by taking the viscoelastic parameters as functions of the stresses being constant for monotonic creep processes. The generalization from this uniaxial model to a fully three-dimensional one is done by means of a projection technique, which gives general representations of tensor-functions identically satisfying the anisotropy of a cubic crystal. The nonlinearity with respect to the stresses is implemented in terms of an irreducible integrity base for the crystal class.

However, this approach is restricted to the undamaged material behavior and converges to the steady-state creep behavior. For higher creep strains, it has to be extended to include the tertiary creep. Microscopic investigations (see Rumi et al. [1994]) show that growth and coalescence of initial microcracks from casting pores are the main reasons for material degradation in both single and polycrystalline superalloys. Therefore, it is reasonable to apply the concept of effective stress from continuum damage theory (see Kachanov [1986]).

**PROCEDURE, RESULTS AND DISCUSSION:** It is widely recognized that the damage process in metals is generally anisotropic, even if the material is initially. Therefore, for single crystals, both the *initial* material anisotropy and the *induced anisotropy* due to damage must be taken into account. Because of this anisotropic nature, the damage influence cannot be represented by a scalar variable alone. An appropriate description of anisotropic damage generally requires a tensorial variable of order two or even higher order. Higher order damage tensors possibly capture the effects of damage more exactly than the lower ones, but it will also take much more effort for the identification and consumes more computation time. Murakami and Ohno [1981] suggested a second-order damage tensor representation. In the present model, such a tensor  $D$  is also chosen, however it appears in the stress equation in a different way, which is motivated from other branches of material modelling. The effective stress  $T_e$ , which appears in the creep model, is defined similarly to the second Piola-Kirchhoff stress tensor  $T$  by

$$T_e = (I - D)^{-1/2} T (I - D)^{-1/2 T},$$

$I$  being the identity tensor, and  $T$  the Cauchy stress (see Codebois, Sidoroff [1982]). For the representation of microcrack opening and closing mechanisms, strain-based projection operators are used to define the active damage tensor and to deactivate the damage effects in the damage tensor under closing conditions.

The experimental investigation shows that the principal tensile stress is responsible for micro crack growth. In isotropic materials, this growth can be expected to be perpendicular to the principal tensile stress direction. For cubic single crystals, similar mechanisms are also observed (see Rumi et al. [1994]). We assume that the damage evolution depends upon the principal tensile stress and its relative orientation with respect to the material structure. Thus, motivated by the Kachanov-Rabotnov theory for the uniaxial loading case, a damage driving force (thermodynamical force associated with damage) is introduced with respect to material anisotropy. The damage law is constructed under consideration of material anisotropy and under thermodynamical restrictions. The damage driving force  $Y_D$  is assumed to be only dependent on the current state of stress and damage, represented by a damage active stress tensor  $S_a$

$$S_a = (I - D)^p T (I - D)^p T = \sum_{i=1}^3 \sigma_i n_i \otimes n_i,$$

according to

$$Y_D = \sum_{i=1}^3 (\eta_i \sigma_i)^k n_i \otimes n_i,$$

where  $\eta_i$  is a function of the relative orientation of the proper vectors  $n_i$  of  $S_a$ ,  $\sigma_i$  its proper values,  $k$  and  $p$  material konstants. The evolution law for the damage tensor is taken as

$$D \cdot = A[Y_D].$$

The fourth-rank tensor  $A$  is constructed under consideration of the material anisotropy and thermodynamical restrictions. In the case of uniaxial loading in lattice direction, the present model coincides with the Kachanov-Rabotnov model.

This model has been implemented into a three-dimensional viscoplastic model and applied to simulation of creep behaviour of single crystal superalloys SRR99 and CMSX-6 in different orientations. It is capable of describing the orientation dependence of the damage behavior as well as the strong non-linearity between creep-strain and creep-stress. It can be used for lifetime-prediction under monotonic creep conditions.

## REFERENCES

- 1986 Kachanov, L.M., Introduction to Continuum Damage Mechanics, Kluwer Acad. Pub., Dordrecht.
- 1981 Murakami, S., Ohno, N., in Ponter, A.R.S., Hayhurst, D.R. ( eds.), Creep in Structures, Springer, Berlin , pp. 422-444.
- 1982 Codebois, J.P., Sidoroff, F., in Boehler, J.-P. ( ed.) Mechanical Behaviour of Anisotropic Solids, Martinus Nijhoff, The Hague , pp.761-774.
- 1994 Rumi, M., Chen, W., Wever, H., Mukherji, D., Kuttner, T., Wahi, R.P., in Coutsouradis, D. et al. (eds.), Materials for Advanced Power Engineering, Part II, Kluwer Acad. Pub., pp.1165-1173.
- 1996 Bertram, A., Olschewski, J., Comp. Mat. Sci. **5**, 12.