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CONSTITUTIVE MODELLING OF PRIMARY AND SECONDARY CREEP OF SINGLE CRYSTALS AT HIGH TEMPERATURES

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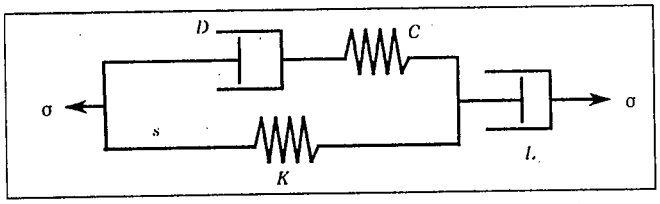
Introduction

Among the great amount of creep theories, few attempts can be found that describe the three dimensional behavior of anisotropic solids like nickel-base superalloy single crystals with f.c.c. structure. The suggestions in this field can be roughly divided into two groups. First, 1-d constitutive creep laws of the octahedral and cubic crystallografic slip systems are composed to a three dimensional (uncoupled) model (/1/-/7/). And, second, the cubic representations of tensor functions and ordered expansions are used to develop three dimensional laws (/8/-/9/). Our approach lies between the two methods. We assume a constitutive law from linear viscoelasticity theory by means of a rheological model and generalize it in order to describe the three-dimensional behavior by a complete cubic representation. The result is a twelve parameter model. A material identification for special orientations was done by an optimization procedure using cyclic tension tests. Thus, we are able to describe the primary and secondary creep behavior and to demonstrate some effects due to the anisotropy.

One dimensional theory

The following rheological model will be used to describe the one dimensional creep behavior of superalloys at high temperatures:



The material constants are the two elasticities C,K and the two viscosities D, L, altogether supposed to be non-negative. The differential equation, which completely describes the behavior of this model (together with initial conditions) is

$$\sigma'' + \left| \frac{C}{D} + \frac{C}{L} + \frac{K}{L} \right| \sigma' + \frac{CK}{DL}\,\sigma = (C+K)\,\varepsilon'' + \frac{CK}{D}\,\varepsilon' \tag{1}$$

which can be integrated in order to obtain an explicit version for the rate of deformations

$$\varepsilon(t)' = \left| \frac{1}{L} + \frac{1}{D}\left(\frac{C}{C+K}\right)^2 \right| \sigma(t) + \frac{1}{C+K}\,\sigma(t)' - \frac{K}{D^2} \left| \frac{C}{(C+K)} \right|^3 \int_0^t \sigma(\tau)\, \exp\left| \frac{CK/D}{C+K}\,(\tau - t) \right| d\tau \tag{2}$$

if the initial state is stress-free. We obtain a more practical form, which is appropriate for computational purposes, if we introduce the force in the lower branch of the model as an internal variable

$$s(t) := K \int_0^t \left| \varepsilon(t)^{\cdot} - \frac{\sigma(t)}{L} \right| dt \tag{3}$$

It satisfies the evolution equation

$$s^{\cdot} = \frac{K}{C+K} \left| \sigma^{\cdot} + \frac{C}{D}(\sigma - s) \right| \tag{4}$$

whereas the incremental form of the material law turns out to be

$$\varepsilon^{\cdot} = \frac{1}{C+K} \left| \sigma^{\cdot} + (\frac{C}{D} + \frac{C}{L} + \frac{K}{L})\sigma - \frac{C}{D} s \right| \tag{5}$$

The numerical integration of these equations is depicted in Fig.1 together with the experimental results (dashed line). The material constants are determined by the latter by means of a nonlinear optimization technique.

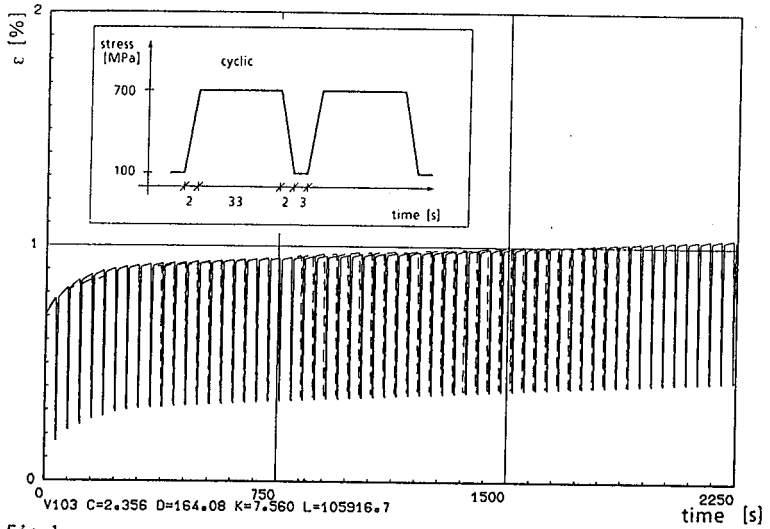


Fig.1
One-dimensional plot of experimental (dashed line) and calculated deformation in cyclic creep test.

Three dimensional constitutive laws

In order to generalize the one-dimensional constitutive equations, we have to satisfy the symmetry properties of the cubic-anisotropic single crystal. The pure elastic (HOOKE's) law of cubic symmetry can always be brought into the form

$$\mathbf{T} = c_1 \, tr(\mathbf{E}_1)\, \mathbf{I} + c_2 \mathbf{E}_2 + c_3 \mathbf{E}_3 \tag{6}$$

with the CAUCHY stress tensor $\mathbf{T}$, the identity tensor $\mathbf{I}$, and the linear deformation tensor $\mathbf{E} := \frac{1}{2}(Grad\,\mathbf{u} + Grad^t\mathbf{u})$, which is decomposed into

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 + \mathbf{E}_3 \tag{7}$$

according to

$$\mathbf{E}_1 := \tfrac{1}{3} tr(\mathbf{E}_1)\, \mathbf{I} \quad ; \quad \mathbf{E}_3 := \sum e_{ii}\, \mathbf{e}_i \otimes \mathbf{e}_i - \mathbf{E}_1 \quad ; \quad \mathbf{E}_2 := \mathbf{E} - \mathbf{E}_1 - \mathbf{E}_3 \tag{8}$$

(summation over the three crystal directions). This decomposition is orthogonal:

$$tr\,(\mathbf{E}_i\,\mathbf{E}_j) = 0 \quad \text{for } i \neq j. \tag{9}$$

If we introduce the linear projection tensors of fourth order

$$\mathbf{P}_1 = \tfrac{1}{3}\mathbf{I}\otimes\mathbf{I}\ ;\ \mathbf{P}_2 = \mathbf{I} - \Sigma\, \mathbf{e}_i\otimes\mathbf{e}_i\otimes\mathbf{e}_i\otimes\mathbf{e}_i\ ;\ \mathbf{P}_3 = \Sigma\, \mathbf{e}_i\otimes\mathbf{e}_i\otimes\mathbf{e}_i\otimes\mathbf{e}_i - \tfrac{1}{3}\mathbf{I}\otimes\mathbf{I} \tag{10}$$

we obtain the comprehensive relation

$$\mathbf{E}_i = \mathbf{P}_i[\mathbf{E}]\ ,\ i=1,2,3 \tag{11}$$

and we can define the fourth order elasticity tensor

$$\mathbf{C} := 3c_1\,\mathbf{P}_1 + c_2\,\mathbf{P}_2 + c_3\,\mathbf{P}_3 \tag{12}$$

such that

$$\mathbf{T} = \mathbf{C}[\mathbf{E}] \tag{13}$$

holds. This representation is the most general of a linear mapping which identically fullfills the cubic symmetry properties and, hence, will be used in what follows in order to generalize the viscoelastic differential equation (2):

$$\mathbf{T}^{\cdot\cdot} + \mathbf{C}_1[\mathbf{T}^{\cdot}] + \mathbf{C}_2[\mathbf{T}] = \mathbf{C}_3[\mathbf{E}^{\cdot\cdot}] + \mathbf{C}_4[\mathbf{E}^{\cdot}] \tag{14}$$

so that each of the fourth order tensors $\mathbf{C}_i$ has the same form as the elasticity tensor (12) before

$$\mathbf{C}_i := 3a_{i1}\,\mathbf{P}_1 + a_{i2}\,\mathbf{P}_2 + a_{i3}\,\mathbf{P}_3 \tag{15}$$

with

$$a_{1j} := C_j/D_j + C_j/L_j + K_j/L_j\ ;\ a_{2j} := C_j K_j / D_j L_j \tag{16}$$

$$a_{3j} := C_j + K_j\ ;\ a_{4j} := C_j K_j / D_j\ .$$

By introducing the tensor of the internal variables like (3)

$$\mathbf{S} := \Sigma_i^3\, K_i\,\mathbf{P}_i \int_0^t \{\,(\mathbf{E}(t)^{\cdot} - 1/L_i\,\mathbf{T}(t))\,\}\,dt \tag{17}$$

we obtain the incremental forms in analogy to equs. (4), (5)

$$\mathbf{E}^{\cdot} = \left(\Sigma_i^3\, 1/(C_i + K_i)\,\mathbf{P}_i\right)[\mathbf{T}^{\cdot}] + \left(\Sigma_i^3\, a_{1i}/(C_i + K_i)\,\mathbf{P}_i\right)[\mathbf{T}] - \left(\Sigma_i^3\, C_i/D_i(C_i + K_i)\,\mathbf{P}_i\right)[\mathbf{S}]$$

$$\mathbf{S}^{\cdot} = \left(\Sigma_i^3\, K_i/(C_i + K_i)\,\mathbf{P}_i\right)[\mathbf{T}^{\cdot}] + \left(\Sigma_i^3\, (C_i/D_i)\,K_i/(C_i + K_i)\,\mathbf{P}_i\right)[\mathbf{T} - \mathbf{S}] \tag{18}$$

which are more convenient for numerical purposes. They contain the 12 material constants C_i , K_i, L_i, D_i, i=1,...,3. If they are all non-negative, then by means of the basic rheological model, compatibility with the second law of thermodynamics is assured. They can be determined by experimental results of samples with different orientations.

The 3-d version of the proposed constitutive law (18) has been implemented in ADINA /10/. The material constants used in the finite element analysis are given in the table.

i	C_i [GPa]	K_i [GPa]	D_i [GPa s]	L_i [GPa s]
1	81.25	250	5 Mio	5 Mio
2	70.00	210	10 000	150 000
3	25.00	75	10 000	150 000

In Fig. 2 we plotted the results of the computer-simulation of creep-tests under constant loads. Fig. 3 gives the first interval of a cyclic creep test. The plots of a cylindrical bar of [0 0 1] and [0 1 2] orientation under tension performes the well-known phenomena due to anisotropy such as tension-torsion coupling, and ovaluation of the cross section (Fig. 4).

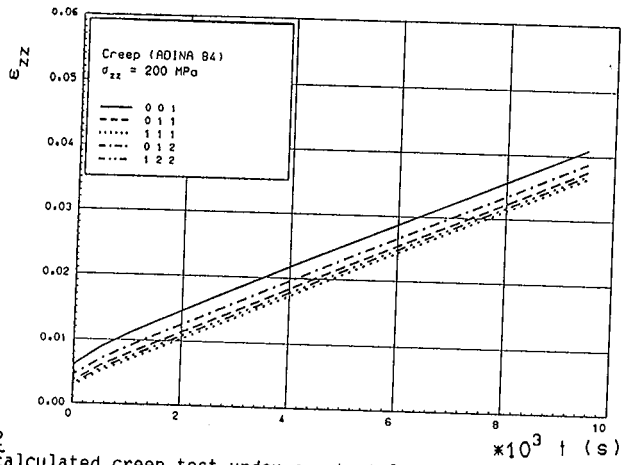


Fig.2
FEM-calculated creep test under constant load in different orientations.

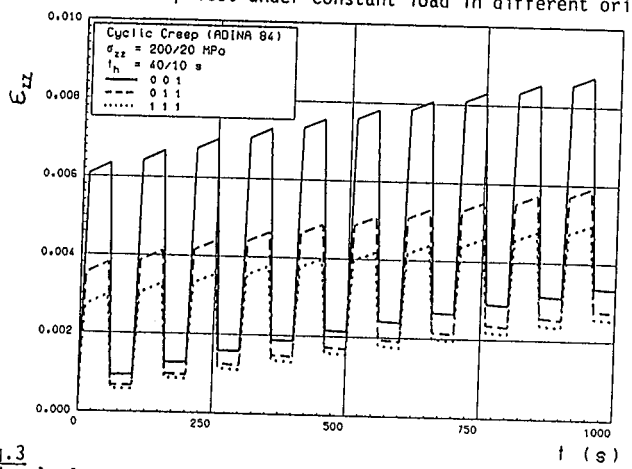


Fig.3
FEM-calculated cyclic creep test in different orientations.

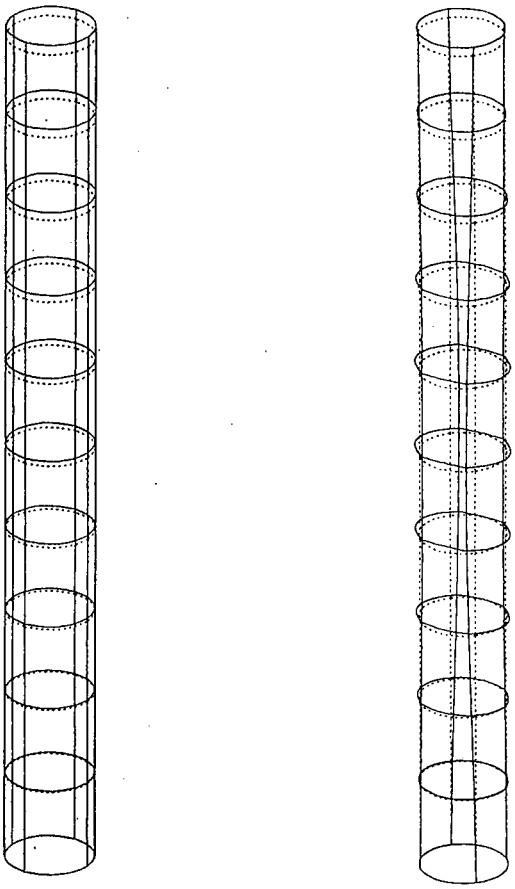
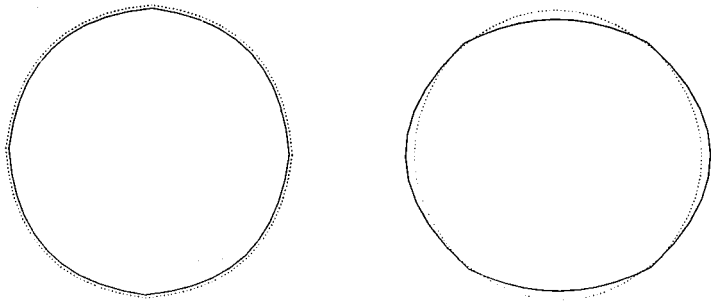


Fig.4
Tension of cylindrical bars (above), their cross sections (below) in [0 0 1] (left) and [0 1 2] (right) orientations.



Conclusions

The suggested constitutive equations describe the general three-dimensional cubic behavior of the model. Its remaining 12 non-negative constants are to be determined by experimental results in different crystallografic directions. The suggestion of composing a linear viscous behavior on all the octahedral and cubic slip systems is included here as a special case.

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