SECTION 5.2

A Phenomenological Anisotropic Creep Model for Cubic Single Crystals

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5.2.1 VALIDITY

The model describes the behavior of cubic single-crystalline superalloys in the high-temperature regime under monotonous creep conditions in their primary and secondary creep phase.

5.2.2 BACKGROUND

For many technical applications, especially in the high temperature regime, single-crystalline components show an improved damage and fatigue resistance in comparison to polycrystals. However, the description of the thermomechanical behavior becomes much more complicated for single crystals because of their genuine anisotropy. The creep compliance for some superalloys differs by one or two orders of magnitude with respect to their

Handbook of Materials Behavior Models. ISBN 0-12-443341-3.

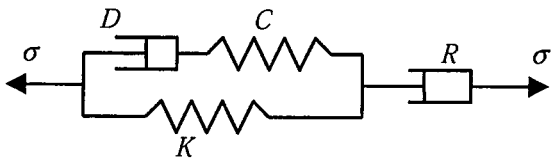

FIGURE 5.2.1

crystallographic orientation. The designer has to take into account this effect, as well as the strong nonlinearity of the relation between force and creep rate.

For the creep modeling of single crystals two conceptually different approaches exist. One is to use slip system kinematics and one-dimensional creep equations for each of them. The other is phenomenological and based on tensor function representations for the description of the anisotropy. The latter approach has the advantage of being simpler in numerical implementation and wider with respect to the physical creep mechanisms to be included in the description, since they are not limited to slip system–dominated creep.

The creep behavior in its primary and secondary phase can be described by a three-dimensional and nonlinear generalization of a rheological model with two springs and two dampers (Fig. 5.2.1).

The one-dimensional constitutive law is given by the ordinary differential equation

$$\ddot{\sigma} + \left(\frac{C}{D} + \frac{K}{R} + \frac{C}{R}\right)\dot{\sigma} + \frac{CK}{DR}\sigma = \frac{CK}{D}\dot{\varepsilon} + (C+K)\ddot{\varepsilon}$$

where ε is the strain, σ is the stress, C and K are elasticities, and D and R are viscosities. The nonlinearity is taken into account by the dependence of the viscosities on the stresses in the following form:

$$D = D_o \exp(-B\sigma)$$

$$R = R_o \exp(-B\sigma)$$

with positive material constants D_o, R_o, and B. Note that D and R are constant during monotonous creep tests.

5.2.3 DESCRIPTION OF THE MODEL

In the three-dimensional case we generalize the preceding differential equation by means of the projection method described in References [1, 2].

This results in three tensor-valued equations

$$\mathrm{T}_i^{\bullet\bullet} + \left(\frac{C^i}{D^i} + \frac{K^i}{R^i} + \frac{C^i}{R^i}\right)\mathrm{T}_i^{\bullet} + \frac{C^iK^i}{D^iR^i}\mathrm{T}_i = \frac{C^iK^i}{D^i}\mathrm{E}_i^{\bullet} + (C^i + K^i)\mathrm{E}_i^{\bullet\bullet}, i = 1, 2, 3$$

with the three cubic subspace projections of the strain tensor ε_{ij} with respect to a crystallographic vector base:

$$\mathrm{E}_1 = 1/3\, tr(\mathrm{E}) \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathrm{E}_2 = \begin{bmatrix} \varepsilon_{11} & 0 & 0 \\ 0 & \varepsilon_{22} & 0 \\ 0 & 0 & \varepsilon_{33} \end{bmatrix} - 1/3\, tr(\mathrm{E}) \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathrm{E}_3 = \begin{bmatrix} 0 & \varepsilon_{12} & \varepsilon_{13} \\ \varepsilon_{12} & 0 & \varepsilon_{23} \\ \varepsilon_{13} & \varepsilon_{23} & 0 \end{bmatrix}$$

with $tr(\mathrm{E}) = \varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33}$. The projections T_i of the stresses σ_{ij} are obtained in an analogous form.

These differential equations of second order can be reduced to a system of first order by introducing a stresslike tensor of internal variables S which is decomposed analogously into three parts S_1, S_2, S_3, by

$$\mathrm{S}_i := K^i \int_0^t [\mathrm{E}_i^{\bullet}(s) - \frac{1}{R^i}\mathrm{T}^i(s)]\, \mathrm{ds}$$

We obtain the following evolution equations:

$$\mathrm{E}_i^{\bullet} = \frac{1}{C^i + K^i}\left[\mathrm{T}_i^{\bullet} + \left(\frac{C^i}{D^i} + \frac{K^i}{R^i} + \frac{C^i}{R^i}\right)\mathrm{T}_i - \frac{C^i}{D^i}\mathrm{S}_i\right]$$

$$\mathrm{S}_i^{\bullet} = \frac{K^i}{C^i + K^i}\left[\mathrm{T}_i^{\bullet} + \frac{C^i}{D^i}(\mathrm{T}_i - \mathrm{S}_i)\right]$$

Note that for monotonous creep all terms including $\mathrm{T}_i^{\bullet}$ are zero. This system can be integrated by standard algorithms such as explicit or implicit Euler schemes.

The nonlinearity is again taken into account by the dependence of the viscosities on the stresses. For that purpose we also use the exponential form of the one-dimensional model but substitute the exponent by a linear form of

the nine cubic invariants J_j

$$D_i = D_{oi} \exp(-\alpha_i)$$
$$R_i = R_{oi} \exp(-\alpha_i)$$
$$\alpha_i = \sum_{j=1}^{9} \alpha_{ij} J_j$$

with positive material constants D_{oi}, R_{oi}, α_{ij}, $i = 1,2,3, j = 1,\ldots,9$. It turns out that the following invariants already give a sufficient accuracy:

$$J_2 = \sigma_{11}\sigma_{12} - \sigma_{12}^2 + \sigma_{11}\sigma_{33} - \sigma_{13}^2 + \sigma_{22}\sigma_{33} - \sigma_{23}^2$$

$$J_4 = \sigma_{12}^2 + \sigma_{23}^2 + \sigma_{13}^2$$

$$J_5 = \sigma_{12}\sigma_{23}\sigma_{13}$$

$$J_6 = (\sigma_{11} + \sigma_{22})\sigma_{12}^2 + (\sigma_{11} + \sigma_{33})\sigma_{13}^2 + (\sigma_{22} + \sigma_{33})\sigma_{23}^2$$

all together taken with respect to a crystallographic basis.

5.2.4 IDENTIFICATION OF THE MATERIAL PARAMETERS

In the present model, the following material constants appear:

- Elasticities: C_i, K_i, $i = 1,2,3$
- Viscosities: D_{oi}, R_{oi}, α_{ij}, $i = 2,3$, j=2, 4, 5, 6

If we assume that all inelastic deformations are isochoric, we get

$$D_1^{-1} = R_1^{-1} = 0 \Rightarrow D_{o1}^{-1} = R_{o1}^{-1} = 0.$$

Therefore, the coefficients $\alpha_1 j$, $j = 1,\ldots,9$ are irrelevant and can be set equal to *1*.

When subjecting the material to resonance tests, the dynamic elastic constants $C_i + K_i$, $i = 1,2,3$, can be determined [5,6]. The other constants can be identified by minimizing the distance between experimental tests and the response calculated by the model. It is desirable for this purpose to have creep data within the relevant load range from single crystals for a widely distributed set of orientations. This leads to an inverse problem of nonlinear parameter identification. The identification has been done for three superalloys: SRR99, CMSX4, and CMSX6.

TABLE 5.2.1 TABLE of Parameters for SRR99 at a Temperature of 760°C [3].

$C_1 = 0$	$C_2 = 42.559$ GPa	$C_3 = 46.764$ GPa
$K_1 = 461.153$ GPa	$K_2 = 28.071$ GPa	$K_3 = 156.436$ GPa
$D_1 = 10^{20}$ h GPa	$D_2 = 2.5042\ 10^{7}$ h GPa	$D_3 = 1.10735\ 10^{9}$ h GPa
$R_1 = 10^{20}$ h GPa	$R_2 = 1.4714\ 10^{9}$ h GPa	$R_3 = 374842$ h GPa
$\alpha_{22} = 12.4347\ \text{GPa}^{-1}$	$\alpha_{32} = 0.24928\ \text{GPa}^{-1}$	
$\alpha_{24} = 1.7874\ 10^{-3}\ \text{GPa}^{-2}$	$\alpha_{34} = 41.783\ \text{GPa}^{-2}$	
$\alpha_{25} = 4.1673\ 10^{-2}\ \text{GPa}^{-3}$	$\alpha_{35} = 4.0697\ 10^{-3}\ \text{GPa}^{-3}$	
$\alpha_{26} = 3.1164\ 10^{-4}\ \text{GPa}^{-3}$	$\alpha_{36} = 5.0154\ \text{GPa}^{-3}$	

Applications to the superalloy CMSX6 can be found in Reference [4].

5.2.5 HOW TO USE THE MODEL

The entire model is given with respect to a crystallographic basis. It is capable of describing the creep behavior under monotonous loads in its primary and secondary phases. There exist extensions of the model to include the tertiary creep phase (damage) [7–9].

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Handbook of Materials Behavior Models

VOLUME I Deformations of Materials

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ACADEMIC PRESS

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London Sydney Tokyo