

Methods to Incorporate Foundation Elasticities in Rotordynamic Calculations

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ABSTRACT

Currently the dynamic effects of foundation are usually neglected for standard design of rotor trains for steam turbines for power generation. Foundation and rotordynamic analyses are conducted nearly independently of each other. However, due to the demand for more precise rotordynamic calculations it is reasonable to take the elasticity of foundation and bearing housing into account in the calculations.

Including the dynamics of the foundation in the rotordynamic calculations can be achieved by using the substructure/transfer function method. The aim of this method, which has been known for decades, is to separate the system into a main structure and a substructure. The elastic substructure, in this case the foundation or foundation and bearing/bearing housing respectively, is harmonically excited with a unity load in degrees of freedom and frequency range of interest at the coupling nodes to the rotor train. From the response at the coupling nodes, a complex flexibility matrix depending on excitation frequency can be derived. After inversion of the matrix, the real part represents the stiffness, which can be added to the stiffness matrix of the rotor system. The complex part of the matrix can be added analogously to the damping matrix. Since these matrices depend on the excitation frequency, only harmonic analyses can be carried out with this procedure. This approach allows to decentralise an overall project with external partners. Each partner can work independently of each other and the calculation tools used can vary. The foundation analysis can be carried out with any FEM-program, also measured transfer function can be included used for the rotordynamic analysis allowing for a comparison of calculation and measurement results. The rotordynamic calculations can be carried out with typical dynamic calculation tools. Having different models for rotor and foundation allows to design of rotor and foundation much easier as dynamic effects can clearly be related to the analysed structure.

Another method to include the dynamic properties of the foundation is to create a combined rotor-foundation-model. On the one hand, this can be done by a truncated modal reduction of the supporting system. Thus, eigenmodes of interest can be included via a reduced model. On the other hand, a model of the foundation using beam elements is appropriate. With these approaches the differential equations cover the total rotor foundation system, and besides the harmonic analysis, eigenfrequencies and eigenmodes can also be calculated.

Combining rotor and foundation in one model results in a multiplicity of eigenfrequencies. The aim of this analysis is to evaluate these coupled rotor-foundation-eigenfrequencies and to identify critical resonances. It will be shown that the methods shown here are efficient analyses tools for cost-effective design of rotor trains.

KEY WORDS

stiffness, damping, support, steam turbine, rotordynamics, foundation, coupling

1 INTRODUCTION

Rotating machinery in power generation is generally supported on a foundation which is, to a certain extent, flexible in the operating speed range. As a consequence, the dynamic behaviour of the foundation can possibly contribute to the dynamics of the rotating machinery. In the past the influence of the foundation on the accuracy of rotordynamic calculations was considered to be of less importance. Thus, the elasticity of the foundation and bearing pedestal was included by rather simple approximations in the standard calculation process for the design of rotor trains. Even though this is cost-efficient and reasonable while rotor and foundation can be considered to be decoupled, there is a need to analyse the dynamic response of rotor trains more precisely

- due to cost optimisation leading to less rigid foundations,
- due to enhanced requirements for low vibrations,
- due to the influence of the foundation/damping on the stability of vibration modes,
- due to increased demand for higher calculation accuracy,
- for unbalance identification [9] and
- when retrofitting turbo trains on existing and less rigidly designed foundations.

Furthermore, by including the foundation and the damping of foundation the stability of rotor modes can be calculated more accurately [5, 11] thus the design of trains can be less conservatism. Also the forces transmitted can be calculated more accurately, as demonstrated in [6, 10].

In the literature [12] mainly two approaches to include the foundation into the rotordynamic calculations can be found, namely the transfer matrix method [4, 15] and the finite element method (FEM) [14, 16]. Also a combination of both approaches can be found [3, 8].

The transfer matrix method is computationally more efficient and has the advantage that different calculation tools can be used, because tools for foundation calculations have to fulfil requirements other than calculation tools for rotordynamics. Additionally measured transfer functions can be used for the rotordynamic calculations. One disadvantage of this method is that only harmonic analyses such as unbalance response calculations can be carried out.

As foundation models consist in general of a large amount of elements, the finite element method is computationally more costly. However, by using beam elements rather than a complete 3D volume model, these costs can be kept to an acceptable level, see [2, 7]. Furthermore, the modal reduction method can be used to reduce the degrees of freedom to an acceptable level. A significant advantage of these approaches is the possibility to calculate eigenfrequencies and stability limits.

This study shows how the two different but well known techniques can be applied practically to real rotor systems. It can be verified that the methods used are appropriate for standard design of rotor train. The vibration amplitudes of the unbalance responses are used to compare the different technique results. This is reasonable since these values are generally measured at the bearing locations of rotor trains while measurements taken directly at the foundation are rare.

2 THEORETICAL BACKGROUND

2.1 Substructure Method/Transfer Matrix

The implementation of foundations in rotordynamic calculations can be achieved by using dynamic stiffness matrices. The main aspects of these algorithms have been well known for more than 30 years [13], but as the original intention of the method is to reduce model size, it has remained unused due to increasing processing power. The coupling of elastic structures such as foundations to rotordynamic programs opens up an interesting field of application, see e. g. [1].

The algorithm is based on the division of the model into main structure and substructure, whereas the substructure consists of the part which is to be reduced for further computation. In the final model used for the computation, the substructure is taken into account at the coupling nodes of the main model.

In general main structure and substructure can be arbitrarily complex. Figure 1 shows a simple example for a system divided into main structure and substructure.

For harmonic response analyses the particular solution can be written in the form of:

$$\mathbf{K}(\Omega)\hat{\mathbf{u}} = \hat{\mathbf{f}} \quad (1)$$

with:

\mathbf{K} - complex dynamic stiffness matrix

$\hat{\mathbf{u}}$ - complex vector of amplitudes

$\hat{\mathbf{f}}$ - complex vector of force

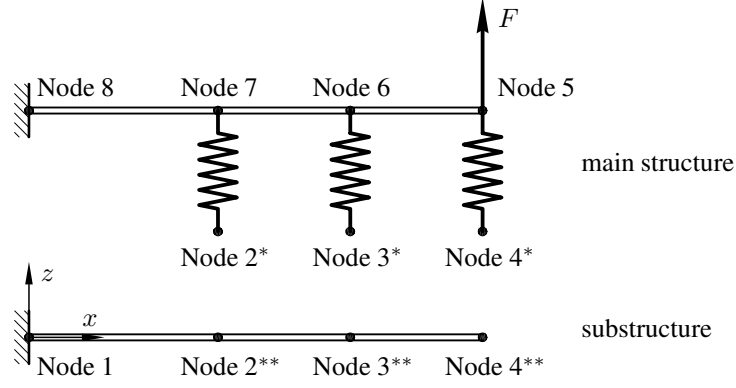


Figure 1: Test model: main structure and substructure coupled by springs.

Equation (1) can be separated into one equation for all nodes of the main structure (index M)

$$\begin{bmatrix} \mathbf{K}_{MM} & \mathbf{K}_{MC} \\ \mathbf{K}_{CM} & \mathbf{K}_{CC} \end{bmatrix} \begin{bmatrix} \hat{\mathbf{u}}_M \\ \hat{\mathbf{u}}_C^* \end{bmatrix} = \begin{bmatrix} \hat{\mathbf{f}}_M \\ \hat{\mathbf{f}}_C^* \end{bmatrix}, \quad (2)$$

and one for the substructure (index S)

$$\begin{bmatrix} \mathbf{K}_{SS} & \mathbf{K}_{SC} \\ \mathbf{K}_{CS} & \mathbf{K}_{CC} \end{bmatrix} \begin{bmatrix} \hat{\mathbf{u}}_S \\ \hat{\mathbf{u}}_C^{**} \end{bmatrix} = \begin{bmatrix} \hat{\mathbf{f}}_S \\ \hat{\mathbf{f}}_C^{**} \end{bmatrix}, \quad (3)$$

where the index C stands for coupling points. Due to the coupling between main structure and substructure, the compatibility conditions

$$\begin{aligned} \hat{\mathbf{u}}_C^* &= \hat{\mathbf{u}}_C^{**} \\ \hat{\mathbf{f}}_C^* + \hat{\mathbf{f}}_C^{**} &= \mathbf{0} \end{aligned} \quad (4)$$

have to be ensured. Finally, the resulting system of equations for the reduced structure can be formulated as

$$\begin{bmatrix} \mathbf{K}_{MM} & \mathbf{K}_{MC} \\ \mathbf{K}_{CM} & \mathbf{K}_{CC} + \mathbf{K}_{Sub} \end{bmatrix} \begin{bmatrix} \hat{\mathbf{u}}_M \\ \hat{\mathbf{u}}_C^* \end{bmatrix} = \begin{bmatrix} \hat{\mathbf{f}}_M \\ \mathbf{0} \end{bmatrix}. \quad (5)$$

For the computation of the dynamic stiffness matrix \mathbf{K}_{Sub} , the substructure is excited by a unity load in the frequency range of interest. The directions of the excitation of the substructure are chosen according to degrees of freedom of the corresponding nodes at the main structure. In general these are the directions which are of interest after a reduction of the model.

This analysis leads to a complex-valued flexibility matrix which depends on the excitation frequency. After inverting this matrix, the real part is to be added to the global stiffness and the imaginary part to the global damping matrix at the coupling degrees of freedom.

In case of coupling a foundation to a rotor train, the foundation is reduced to the coupling degrees of freedom between the foundation and the rotor train. Thereafter the resulting dynamic stiffness matrix of the reduced system (here the foundation, which can consist of any type of element) is added to the coupling degrees of freedom of the rotor train. The resulting model has the characteristics of the entire model for the harmonic analyses.

The disadvantage of this method is the dependency of the resulting matrix \mathbf{K}_{Sub} on the excitation frequency. This is why only harmonic analyses can be carried out with this coupling method. In addition, as the substructure is represented by matrices, it is not possible to visualise nodes of the substructure (foundation) other than the coupling nodes to the rotor.

In Figure 2 the vibration amplitudes for node 7 of the model shown in Figure 1 are displayed. The vibration amplitudes when using the substructure, and a complete model are shown in black and grey respectively. Both calculation methods show the same results. Minor differences are due to rounding errors when storing the transfer matrix. The advantages of the transfer function method lie in the reduced simulation time and in the achieved accuracy of the method within the discretisation error. Furthermore, an entire system can be separated into different parts and modelled by several partners regardless of the calculation tool and finite elements used. Additionally, with the transfer function method it is possible to include experimental results in the calculation model, i. e. measured dynamic stiffness properties.

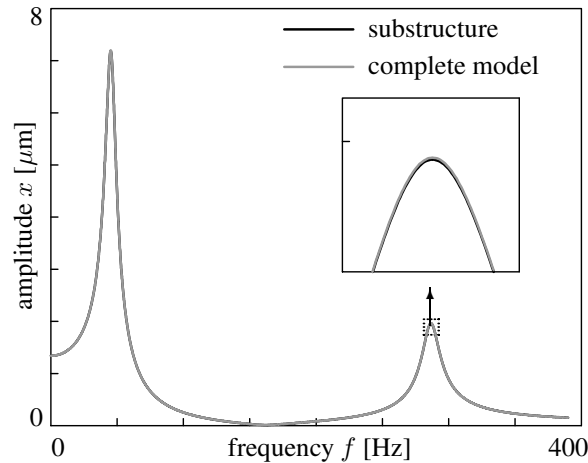


Figure 2: Accuracy of the transfer function method for node 7.

2.2 Complete Rotor-Foundation-Model

Another possibility of coupled calculations of foundations and rotor trains is the complete implementation of the foundation in the rotordynamic program. To model the foundation mainly beam elements are used, which are also available in usual rotordynamic programs. Rather than using a reduced model for the foundation, here the focus is only on foundation models using beam elements, as these are used in the design phase of foundations and thus easily available.

Since the models of the foundation and the rotor train are generated independently and with different programs, an interface had to be developed. This interface is needed for the conversion of the foundation's FE input data into the rotordynamic program.

In general, the definition of the FE-model (nodes, elements or material properties) is introduced by several keywords. The algorithm of the interface consists of two parts. The first part, which depends on the FE-program used for the modelling of the foundation, searches the input files for the defined keywords. The essential information is stored in a neutral format. The second part uses this neutral format and generates an additional input file of the foundation model for the rotordynamic program. As the neutral format is program independent, the second part is identical for every foundation model. The neutral format is schematically given in Table 1.

Table 1: Neutral format for data storage.

array name	description
nodes	node number, coordinates, degrees of freedom
elem	element number, topology (nodes of the element), element type, real set, material
real set	real set number, parameters (depending on the element type)
type	type number, additional parameters (e.g. spring direction)
mat	mat number, Young's modulus, Poisson's ratio, density, damping

3 SINGLE ROTOR IN BALANCE SHOP

As an application for the transfer function method measurements taken at the balance shop are used. Figure 3 shows a low pressure (LP) turbine on the pedestals used in the balance shop. As the vibration amplitudes of the pedestals are used for balancing, the pedestals can be considered to be elastic. In the balance shop, the bearing pedestals are relocated from the carrier train to the foundation of the balance shop. At each bearing pedestal there are two vibration sensors at an angle of 45° from the vertical axis.

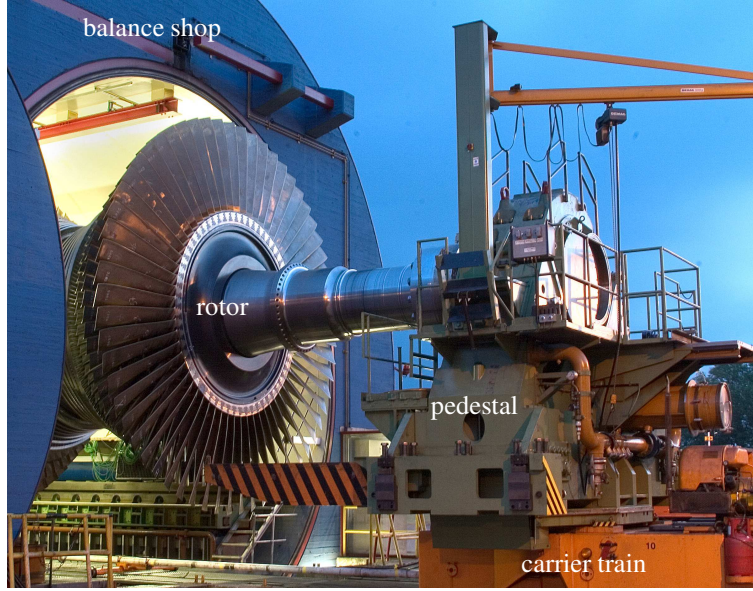


Figure 3: Low pressure rotor and balance shop.

3.1 Measured Transfer Function

The sensors at the bearing pedestal are calibrated on a regular basis. In order to do this, an unbalance exciter is placed directly into the bearing. As unbalance and speed of the exciter are known, the exciting force can be calculated. As a secondary effect a transfer function

$$H_{m,n}^{i,j}(\Omega) = \frac{X_m^i(\Omega)}{P_n^j(\Omega)} \quad m, n, i, j = 1 \dots 2 \quad (6)$$

can be extracted from the measured response X and the calculated exciting force P due to the unbalance excitation. In the above equation the subscripts m, n specify the direction of excitation, the superscripts i, j the bearing number.

By inverting the transfer matrix, the dynamic stiffness \mathbf{H}^{-1} of the bearing pedestal can be calculated. As the foundation in the balance shop is rather rigid compared to the bearing pedestal, no cross coupling between the two supports occurs in the frequency range of interest. Therefore, in the transfer matrix the entries $H_{m,n}^{i,j}$ where $i \neq j$ are zero. The excitation mechanism has the disadvantage that at the same time the horizontal and vertical direction are excited. Hence, it is impossible to distinguish if the response X in one direction is due to the excitation P in the orthogonal direction. Thus, it is considered that $H_{m,n}^{i,j} = 0$ for $m \neq n$. As a special case, transfer functions for each bearing pedestal are obtained rather than a transfer matrix.

The stiffness matrix \mathbf{K} of the support structure is obtained by inverting the transfer matrix \mathbf{H} ,

$$\mathbf{K} = \mathbf{H}^{-1}. \quad (7)$$

Figure 4 shows measured speed dependent stiffness/damping properties of bearing pedestals used in the balance shop. The black lines show the vertical, the grey ones the horizontal stiffness obtained by the transfer function method. For high excitation frequencies/rotor speeds the dynamic stiffness coefficients become significantly

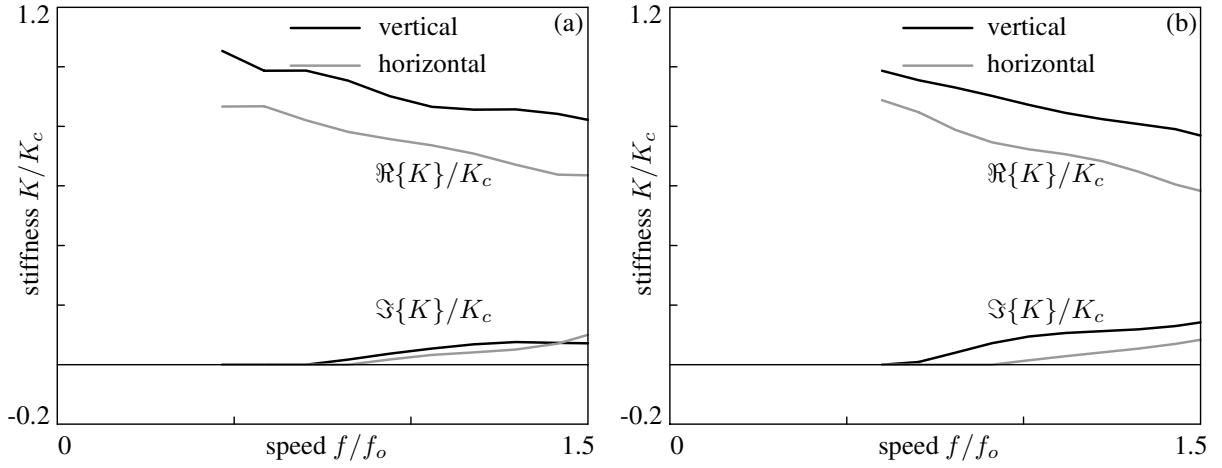


Figure 4: Measured dynamic stiffness of supporting structure in the balance shop. (a) drive end, (b) free end.

smaller and might significantly influence the calculated eigenfrequencies. Additionally, damping coefficients unequal zero appear, influencing the vibration amplitudes of the unbalance response.

3.2 Unbalance Response

Table 2 shows the measured and calculated eigenfrequencies of an LP rotor in the balance shop. The resonance speeds of the measured unbalance response matches perfectly with the resonance speeds obtained by using the transfer function method.

Table 2: Comparison of measured and calculated vertical eigenfrequencies f/f_o .

Mode Shape	Measurement	Transfer Functions
1.	0.39	0.40
2.	1.08	1.08

For balancing the rotor train only the vertical component of the two vibration sensors are used. In Figure 5 the vibration amplitudes of the LP rotor are compared with the calculated ones using the transfer function method. The unbalances used for the calculation of the unbalance response are obtained by an unbalance identification method described in [9]. Especially at the free end, where no influence of the driving cardan shaft and motor is present, the courses match very well.

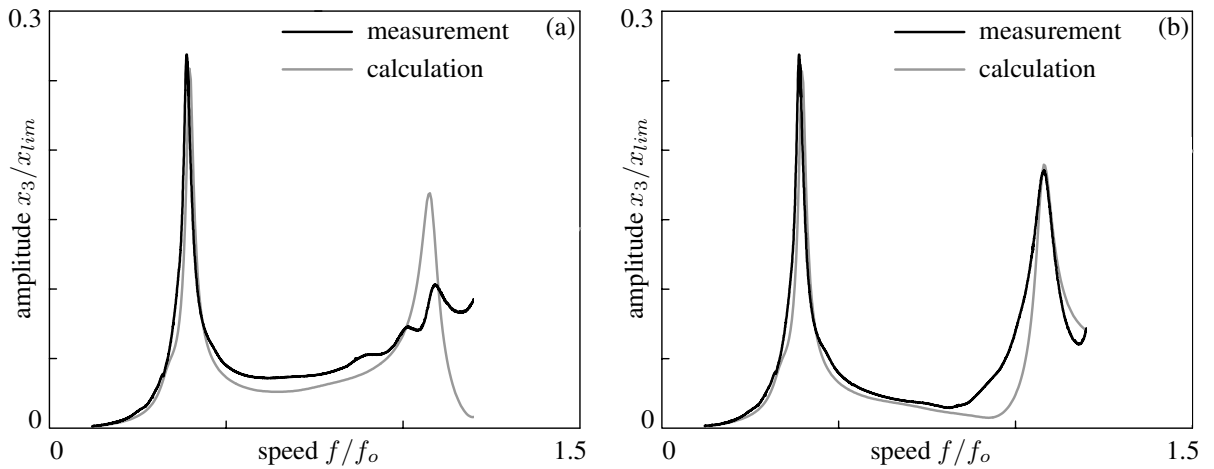


Figure 5: Bearing vibrations as result of unbalance excitation. Vertical amplitude at (a) drive end, (b) free end.

4 TOTAL ROTOR TRAIN IN POWER PLANT

After having verified the method of transfer functions in the balance shop, the next step is to transmit it to a total rotor train. For this purpose several rotor trains are modelled for calculations with both the transfer function method and as a combined rotor-foundation-model.

4.1 Transfer Function Method

A further verification can be carried out by means of vibration measurement at a complete rotor train, consisting of a high pressure (HP), a intermediate pressure (IP), three low pressure (LP) turbines, a generator (Gen) and an exciter (Ex). For the complete rotor train no measured transfer functions/matrix of the supporting structure are available. Thus the transfer matrix is derived from FE-analysis using ABAQUS. As described previously, foundations are mostly modelled with 3D-beam elements. In Figure 6 two foundation models used for the later analysis are shown. Modal damping is assumed to be 2%. Excitation of the coupling nodes in vertical and horizontal direction leads to the required transfer matrix, as described in 2.1.

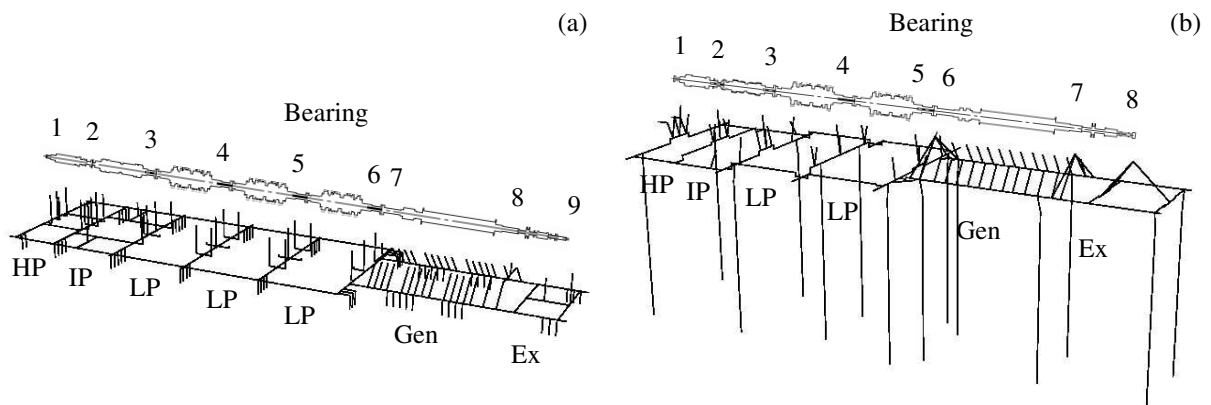


Figure 6: FE-models to calculate transfer matrix for different foundations. (a) foundation model for rotor train with three LP rotors, (b) foundation model for rotor train with two LP rotors.

As a first application for the transfer function method, noticeable vibrations at the bearing between the first and the second LP-rotor (bearing no. 4) during a start-up procedure can be explained. The curve in Figure 7 (a) shows measured bearing vibration amplitudes at the bearing between the first and the second LP-rotor. Resonances can be seen in the speed range around 0.4 and 0.85 times the nominal speed.

Neglecting the influence of the foundation, no elevated vibration amplitudes occur in the range $f \approx 0.85 f_o$, see the grey curve in Figure 7 (b). The black curve shows the bearing vibrations using the transfer matrix calculated

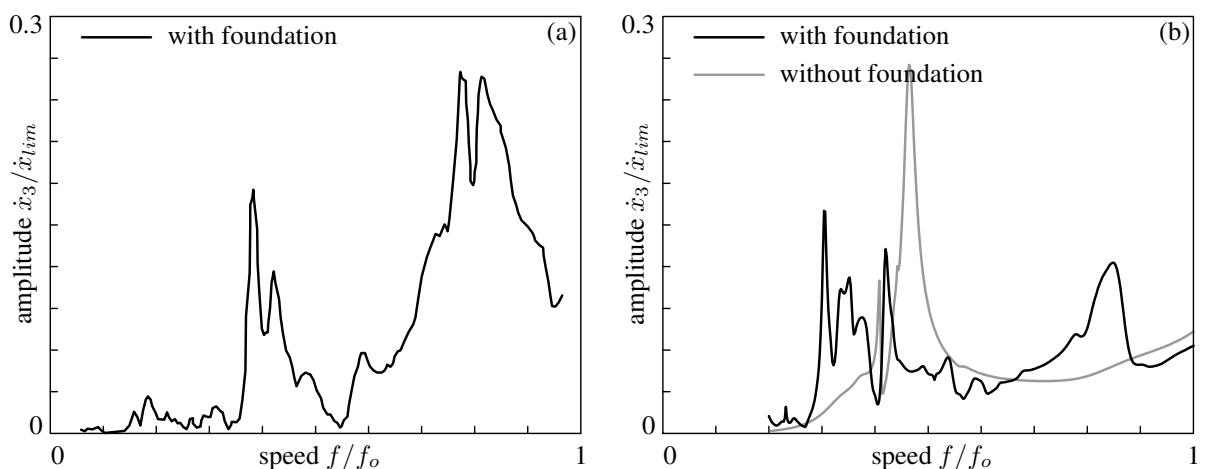


Figure 7: Bearing vibrations at bearing no. 4 as a result of unbalance excitation. Amplitude of major semiaxis from (a) measurement, (b) calculation.

from the FE-model shown in Figure 6 (a). Including speed dependent pedestal properties the resonance can be explained, leading to a similar resonance frequency as in the measurement, see the black curve in Figure 7 (b). Also the frequencies in the range $f \approx 0.4 f_o$ match much better with the measured ones when using the transfer function method. However, there are still some differences in resonance frequencies visible. They might be due to fact that non-linearities in the contact area between foundation and bearing housing or bearing housing and bearing respectively are not considered in the model.

Note that, the vibration amplitudes for the calculation are based on unbalances placed in such a way that all eigenmodes lying within the operational speed are excited. However, the magnitude of the unbalances used in the calculations does not fit to the unbalance state of the real rotor train. As a consequence, the vibration amplitudes do not match the measured amplitudes.

For the foundation model shown in Figure 6 (b), the influence of the foundation on the unbalance response of the rotor train can be seen in Figures 8 and 9. In this case the rotor train has only two LP rotors. In these figures the calculated vibrations at the bearings located next to the LP rotors are shown. Due to the fact that the same unbalances are applied in both cases, conclusions on the influence of the supporting structure with respect to vibration amplitudes can be made.

In Figure 8 the absolute shaft vibrations are shown. In the range below $f = 0.5 f_o$ the resonance frequencies are reduced by up to 15% but also the vibration amplitudes are smaller when using the transfer matrix. For higher speeds the differences become less significant. In the absolute shaft vibrations no additional resonance frequencies can be found using the transfer function method.

As in Figure 7 (b), the bearing vibration with and without foundation effects show more significant differences.

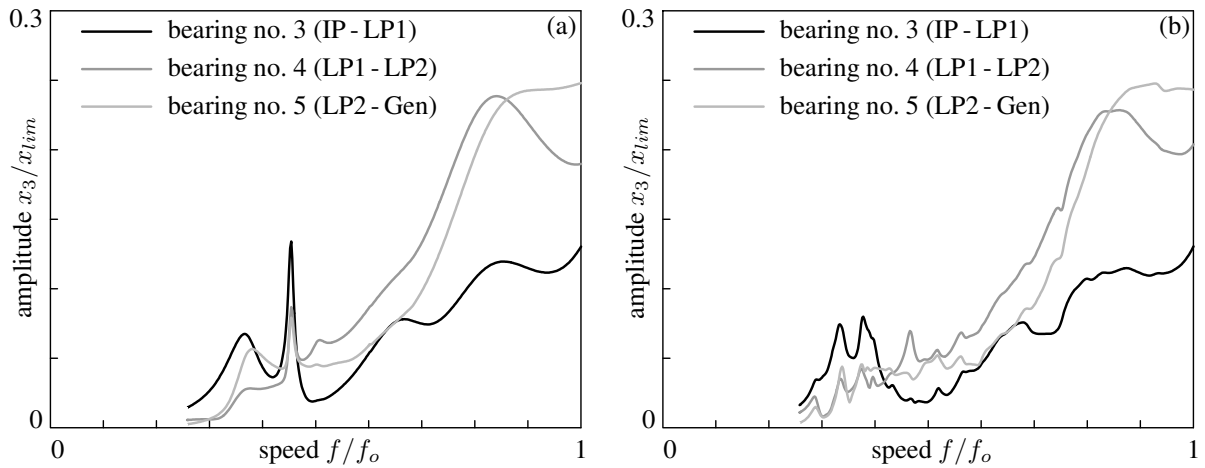


Figure 8: Absolute shaft vibrations as a result of unbalance excitation. Amplitude of semi-major axis (a) without, (b) with transfer function method.

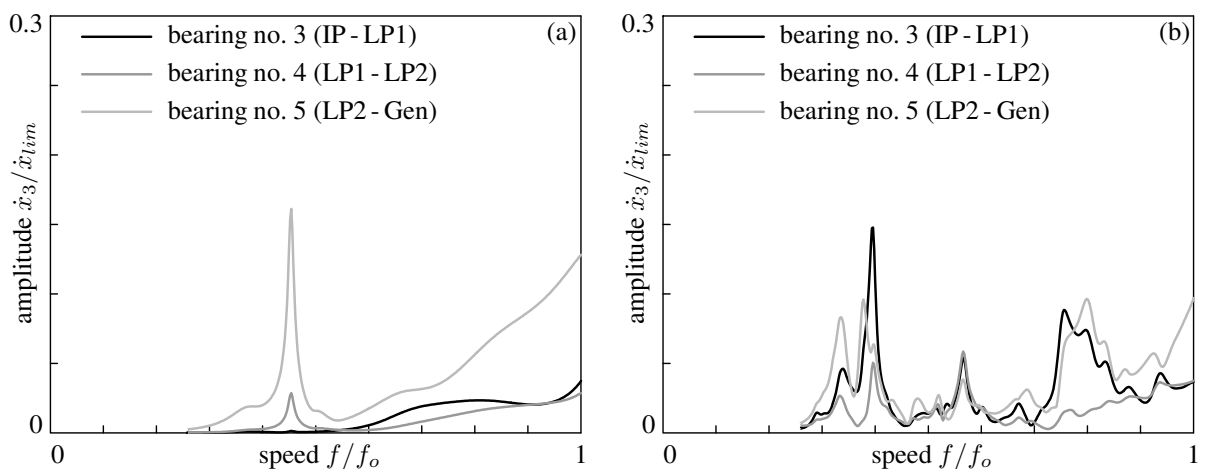


Figure 9: Bearing vibrations as a result of unbalance excitation. Amplitude of semi-major axis (a) without, (b) with transfer function method.

In Figure 9 (b) additional resonance peaks at $f = 0.57 f_o$ and in the range around $f \approx 0.8 f_o$ are visible. However, for the foundation model shown in Figure 6 (b) the influence of the supporting structure on the bearing vibration is much less distinct compared to the influence on the foundation shown in Figure 6 (a).

In general, the damping of the foundation leads on the one hand to lower amplitudes, on the other hand at certain frequencies to higher peaks due to foundation eigenfrequencies or coupled rotor-foundation eigenfrequencies. This means that the influence of the foundation can lead to a positive effect in terms of lower amplitudes, but also to higher peaks or a shift of eigenfrequencies at certain speed ranges. Especially the bearing forces might change significantly by including the foundation in the calculations.

4.2 Combined Rotor-Foundation-Model

The second possibility to consider the foundation in a rotordynamic analysis is a combined rotor-foundation-model. Due to restrictions of the rotordynamic program, this approach is limited to foundations modelled by 3D beam elements. Foundation models obtained by a modal reduction are not included in this analysis. The main advantage of this procedure is the possibility to calculate eigenfrequencies of the entire system. The unbalance response, of course, leads to the same results as the transfer function method as long as the model of the foundation is the same.

By combining rotor and foundation in one complete model, three different eigenmode "types" can be identified:

1. Eigenmodes of the rotor train (affected by the foundation through shift of frequency)
2. Eigenmodes of the foundation
3. Combined rotor-foundation-eigenmodes (interaction between rotor train and foundation)

Particularly the last aspect has not yet been examined. But as coupled eigenfrequencies might occur close to operating speed, this again shows the importance of considering the rotor train together with the foundation.

In Figure 10 a mode shape of the rotor train of the rotor train shown in Figure 6 (b) is shown. In the shown case, the foundation does not participate significantly in the vibration ("type" 1). However, even though the foundation does not participate in the vibration significantly, the considered eigenfrequency of the rotor train is shifted by 3.5%. For the mode shape shown here, the modal damping of the mode shape is 7.7% smaller when including the foundation in the dynamic analysis.

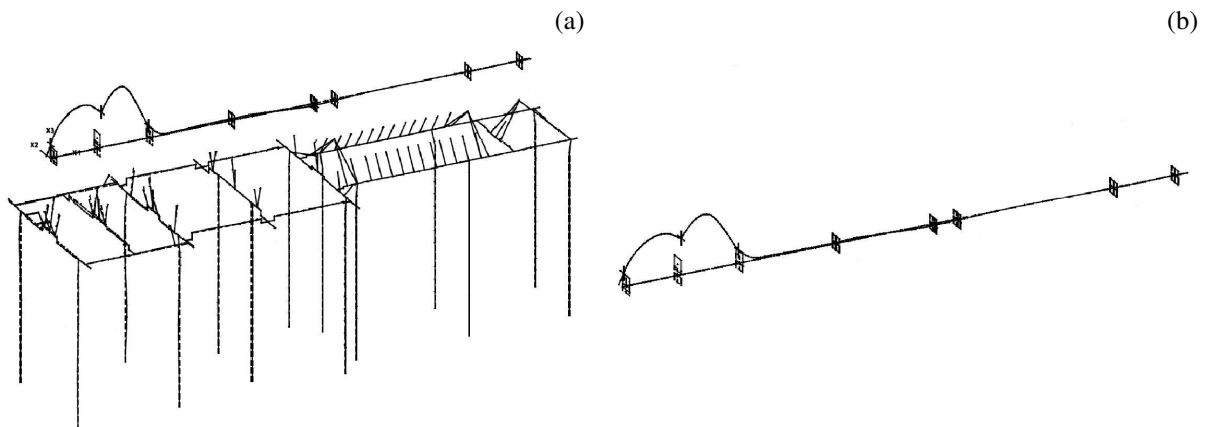


Figure 10: Eigenmodes of a rotor train. (a) with foundation, (b) without foundation.

5 CONCLUSION

The transfer function method is an easy and powerful method for including the foundation/pedestal dynamic characteristics in the rotordynamic calculation. For the bearing pedestals used in the balance shop the resonance frequencies as well as amplitudes in the unbalance response calculation using the transfer function method yields very good results.

For the complex system of a whole rotor train based on a foundation, the transfer function method leads to convincing results concerning frequencies of an unbalance response. By including the foundation, measured resonance frequencies can be explained. Of course, the vibration amplitudes between measurement and calculation differ as

long as different unbalance states are used. Divergent calculation results might be reduced further by considering non-linear effects within the foundation and the area of contact between foundation and bearing components.

For the investigated rotor trains and foundations the absolute shaft vibrations are less affected by including the foundation in the analysis as the bearing pedestals are rather rigidly connected to the foundation. Even though the foundation has more influence on the bearing vibration, depending on rotor and foundation, it might or might not be significant. However, it is reasonable to include the foundation in the design process of rotor trains as bearing forces and stability of eigenmodes can be calculated with more accuracy.

By implementing the foundation in the rotordynamic program, it is not only possible to carry out harmonic analyses but also to determine eigenfrequencies and mode shapes. With these calculation tools beside realistic shaft vibrations also realistic pedestal vibrations can be calculated. The coupling of rotor and foundation in one model results in a multiplicity of eigenfrequencies. It is possible to evaluate these coupled rotor-foundation-eigenfrequencies and critical resonances can be identified by means of unbalance response calculations. With the described methods, calculation tools are available which can easily be used within the standard calculation process of rotor trains.

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