A nonlinear model for rotor-shaft joints of high speed rotor systems with internal damping

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Viscous internal damping in joints of high speed rotor systems causes instabilities above a certain frequency of revolution. In the majority of cases a nonlinearity adjusts the stability margin towards higher frequencies.

In this paper an analytical solution of a nonlinear four degrees of freedom rotor model with internal damping is proposed, which enables to clearly analyse the influence of shaft stiffness, connection stiffness, rotor mass and shaft mass. The steady state solution of the unbalance case and the stability boundaries are deduced analytically. The stabilizing effect of the nonlinearity is shown. The analytical solutions are in good agreement with numerical results obtained by FERAN, a rotor dynamic simulation tool. A model, representing the rotor-shaft connection with an o-ring has been analyzed by a hydro pulse rig. Beneath the linear way, two further approaches to describe the measured hysteresis, a cubic and a bilinear force law are shown in the paper. The different analytical and numerical results for the whole rotor system with these three approaches are compared.

1 Introduction

Rubber elements connecting rotor and shaft of high speed lab centrifuges are sometimes used to uncouple rotor vibrations from shaft movements and to facilitate the change of rotors. Nevertheless these elements induce energy dissipation in the rotating systems, such that instabilities occur above a certain frequency of revolution. In one device, increased but bounded amplitudes have been measured, which could be traced back to internal damping working in an o-ring.

Viscous internal damping can be modelled by a spring damper element between rotor and shaft. If the spring is linear, the effect of bounded amplitudes could not be observed. Hence a non-linear model is necessary. It enables to take into account nonlinearities, for example due to a clearance between rotor and shaft.

Stabilizing effects of nonlinearities in rotor systems with internal damping were shown by [1] on a two degree of freedom model. To allow a clear analysis of the parameters, which influence the stability margin, like shaft and joint stiffness, rotor and shaft mass, a for degree of freedom model should be used. It fits better to applications, where viscous internal damping of the rotor shaft joint dominates and internal damping in the shaft has a minor influence.

2 Steady state solution and stability of a rotor with nonlinear rotor-shaft joint model

The analyzed four degrees of freedom rotor system consists of a shaft with stiffness $k_u$ and mass $m_w$ and a rotor with mass $m_R$. A cubic force law is used to describe the nonlinear joint stiffness:

$$ f = (k_i + k_{inl} |\Delta r|^2)(r_2 e^{i\varphi_2} - r_1 e^{i\varphi_1}). $$

(1)

$k_i$ is the linear stiffness of the rotor-shaft connection, $k_{inl}$ is the nonlinear rotor-shaft-connection stiffness, $r_1$ is the radial deflection of the center of gravity of the shaft and $r_2$ those of the rotor (Fig. 2). The equation of motion is stated in complex form in polar coordinates to enable the calculation of the steady state solution. In the steady state case all time derivatives of the deflections $r_i$ and the angular accelerations vanish. One can deduce a 3rd order polynomial for the relative deflection between rotor and shaft $\Delta r$ in the steady state case:

$$ ((-\frac{m_w}{m_R} + \frac{k_w}{m_R \Omega^2} - 1)k_{inl})\Delta r^3 + (-\frac{m_w}{m_R}k_i + m_w\Omega^2 + \frac{k_w k_i}{m_R \Omega^2} - k_w - k_i)\Delta r - (k_w - m_w \Omega^2)\frac{m_w}{m_R}r_w = 0. $$

(2)

Equation 2 is solved analytically for the case with and without unbalance load $m_w r_w \Omega^2$. The solution for a rotor system without unbalance load is shown in figure 1. Frequencies $\Omega_{1/2}$ separate the frequency areas of trivial and nontrivial solutions.

$$ \Omega_{1/2}^2 = \frac{k_i}{2m_R} + \frac{k_w + k_i}{2m_w} \pm \sqrt{\left( \frac{k_i}{2m_R} + \frac{k_w + k_i}{2m_w} \right)^2 - \frac{k_w k_i}{m_R m_w}} $$

(3)

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The lower frequency $\Omega_1$ corresponds to the boundary frequency of the linear rotor system with internal damping and to the lowest resonance frequency of the linear system (2). The boundary frequency separates the stable from the unstable speed range.

To analyze the stability of the system, the equation of motion is linearized around the steady state solution. The stability margin $\Omega_a = \sqrt{k_w/(m_w + m_R)}$ is deduced with the help of the Thomson and Tait theorem. It follows that in the nonlinear case the stability margin is moved from the frequency $\Omega_1$ towards the frequency $\Omega_a$.

### 3 Example: rotor system with o-ring as rotor-shaft joint

As an example, we consider a rotor system with two o-rings connecting rotor and shaft. The whole system has been analyzed in a hydro puls unit. The displacement between rotor and shaft is plotted in figure 3. The measured hysteresis loop flattens for high displacements as the clearance between rotor and shaft is reached. The results are used to deduce a force law which describes the properties of the connection. The test was simulated numerically. The simulation enables to find appropriate values for the parameters of different force laws.

The use of a linear force law for the o-ring leads to an unacceptable difference between measurement and simulation. A cubic o-ring model enables an analytical solution for the stability boundaries and the steady state deflections of the rotor system. However, this model has some drawbacks. The linear stiffness is too high and the bending of the curve is not reflected very well. For a numerical simulation of the rotor system, a bilinear cubic force law can be used, $F_c = c_i \cdot \Delta x + k_i \cdot \Delta x + k_{ir} \cdot (\Delta x - s) + k_{inl} \cdot \Delta x^3$. A new linear relative stiffness is introduced which is multiplied by the difference between relative displacement and clearance $s$ and which only acts, if the relative displacement is higher than the clearance. The linear relative stiffness is much higher than the linear stiffness. The simulated and measured hysteresis loop coincide well.

Numerical simulations of a run-up of the rotor system are shown in figure 4. There exists only a stabilizing effect, if a nonlinear force law is used. The linear rotor model becomes unstable after passing the linear boundary frequency. The numerical solution with a cubic o-ring model coincides well with the analytical one.

### References