Some considerations of modelling internal friction in rotor-shaft connections

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Abstract—Different rotor systems with internal damping in the rotor shaft connection are investigated in the following. The energy dissipation in the rotating joint is described by viscous internal damping and dry friction. The main influence parameters for the stability of the rotor systems are deduced analytically and numerically.

Keywords: stability, internal damping, internal dry friction

I. Introduction

To ensure a safe operation of high speed lab centrifuges it is necessary to avoid possible sources of instabilities already in the design process. In this context it is necessary to rely on simulation tools, design rules and experience. Instabilities are caused by energy dissipation in the rotating system, especially in the joint of shaft and rotor.

The shaft rotor connection of high speed centrifuges operating with different rotors is designed in such a manner that the connection is easily released for changing the rotor and that the rotor is attached firmly during operation. Nevertheless small motions can occur in-between of rotor and shaft due to incorrect operation, manufacturing errors or constructive errors. Thus damping or friction is inserted in the rotating system which provokes a destabilization in the post critical speed range.

In some cases of instability an enlargement of a rotor due to centrifugal forces was observed especially when the material properties of the rotor differ widely from that of the shaft. Relative movements appear which are difficult to describe.

Rubber elements are sometimes deliberately introduced in the joint of rotor and shaft to facilitate the change of rotors or to uncouple rotor and shaft movements to ease self-centering of the rotor in the post critical speed range. However they introduce internal damping into the rotor system. To avoid instabilities the choice of these rubber elements has to be accomplished carefully.

For simulation, those rubber elements can be qualified as spring-damper elements between rotor and shaft using a viscous damping model. Due to its linear character the implementation of viscous internal damping in simulation models and the subsequent computation is comparatively easy. This paper deals with two different Jeffcott rotors with internal damping to figure out the main influence parameters to the range of stability. In addition a comparison between measurements and simulation results of a rotor-system with rubber elements in the rotor-shaft connection is presented.

Internal dry friction fits more to the micro-movements which occur in-between shaft and rotor. One model for internal dry friction is studied on a Jeffcott-Rotor. The simulation results are included.

The numerical simulations in this paper were executed with the simulation program FERAN. It was developed by the authors for the simulation of rotor-systems [3]. It offers the possibility to calculate eigenvalues, steady state and time-responses, bearing forces and to include internal damping, nonlinearities and journal bearings.

II. Viscous Internal Damping

Viscous internal damping is the easiest way to describe energy dissipation in rotor dynamics. It is commonly used to characterize damping of fluids or viscous dampers.

Prior to examine the implementation of viscous internal damping in the simulation of high speed centrifuges, we first analyze a Jeffcott-rotor with different models of viscous internal damping in order to identify the main parameters influencing the area of stability.

A. Two degrees of freedom Jeffcott Rotor considering internal damping in the shaft

If the material damping in the shaft of a two degrees of freedom Jeffcott rotor is small enough it can be represented approximately by viscous damping. The internal viscous damping is proportionally to the speed of deflection in the rotating coordinate system.

It is possible to calculate the eigenvalues of this system analytically. The system is stable if the real part of all eigenvalues is negative. With some basic assumption one gets the boundary frequency \( \omega_b = 1 + \frac{c_i}{m_R} \) (2). The boundary frequency \( \omega_b \) depends only on the ratio of internal and outer damping. For frequencies higher than \( \omega_b \) instability is to expect.

The dimensionless frequency \( \omega \) is defined as \( w = \frac{\Omega}{\omega_0} \) with \( \Omega \) the speed of revolution and \( \omega_0 \) the eigenfrequency. The damping coefficients are \( \vartheta_i = \frac{c_i}{2\omega_0 m_R} \) and \( \vartheta_a = \frac{c_a}{2\omega_0 m_R} \) with internal damping \( c_i \), outer damping \( c_a \) and rotor mass \( m_R \).

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B. Jeffcott Rotor with internal viscous damping in rotor shaft connection

Internal viscous damping in the connection between rotor and shaft of high speed lab centrifuges occurs when rubber elements are embedded between rotor and shaft. These rubber elements are utilized to uncouple rotor vibrations from the shaft movements, to ease self-centering of the rotor and to facilitate the change of rotors. They can be qualified with spring damper elements between rotor and shaft.

![Figure 1. Jeffcott rotor with internal damping c, stiffness of rotor-shaft connection k_s, shaft stiffness k_w, motor mass m_R, shaft mass m_W and outer damping c_w](image)

To figure out the basic influence of the stiffness, damping and mass parameters we consider a basic Jeffcott-Rotor with one spring damper element between rotor and shaft (figure 1). After some calculations, we get the equation of motion in the reference coordinate system in a non-dimensional form.

\[
[M] \cdot \{x\}'' + [C] \cdot \{x\}' + ([K] + [P]) \cdot \{x\} = [0] 
\]

(1)

with the abrevations

\[
[M] = \begin{bmatrix}
\rho & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 \\
\end{bmatrix},
\quad [A_1] = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 \\
\end{bmatrix},
\quad [A_2] = \begin{bmatrix}
0 & 1 & 0 \\
0 & 0 & 1 \\
1 & 0 & 0 \\
\end{bmatrix},
\quad [B] = \begin{bmatrix}
0 & 0 \\
0 & 0 \\
0 & 0 \\
\end{bmatrix}
\]

the matrices in equation (1) are

\[
[M] = \begin{bmatrix}
[m] & [0] \\
[0] & [m] \\
\end{bmatrix},
\quad [P] = 2w\vartheta \begin{bmatrix}
[0] & [0] \\
[A_1] & [A_1] \\
\end{bmatrix}
\]

\[
[C] = 2 \vartheta [A] + \varepsilon [B],
\quad [K] = [A] + \kappa [B]
\]

The dimensionless entities are mass ratio \( \rho = \frac{m_W}{m_R} \), stiffness ratio \( \kappa = \frac{k_w}{k_s} \), outer damping ratio \( \varepsilon = \frac{c_w}{\omega_0} \), \( \omega_0 = \frac{k_s}{m_R} \), internal damping ratio \( \vartheta = \frac{c_s}{2\omega_0 m_R} \) and frequency ratio \( w = \frac{\Omega}{\omega_0} \). The vector \( \{x\} = \begin{bmatrix} x_w & x_R & y_w & y_R \end{bmatrix}^T \) consists of the dimensionless rotor deflections (index \( R \)) and of the dimensionless shaft deflections (index \( w \)).

To compute the boundary frequency which separates the stable from the unstable speed range, we first assume that there is no outer damping, that means \( \varepsilon = 0 \).

The stability of the equation of motion 1 can be analyzed by an extension of the Thomson and Tait theorem:

*The system*

\[
[M] \cdot \{x\}'' + [C] \cdot \{x\}' + ([K] + [P]) \cdot \{x\} = [0] 
\]

is stable if \([M] \) and \([K] + [P] \) are symmetric and positive definite, \([C] \) is symmetric and positive semi-definite and \([G] \) is a skew-symmetric gyroscopic matrix (11).

Due to the terms of the internal damping, the matrix \([K] + [P] \) is not symmetric such that the equation (1) should be transformed to eliminate the skew symmetric terms in the matrix \([K] + [P] \). Following a proposal of [1] this can be done with a transformation matrix \([L] \), whereby \([L] \) fulfills the differential equation

\[
[C] \cdot [L]'' + [P] \cdot [L] = [0] 
\]

(3)

One obtain

\[
[L] = \begin{bmatrix}
[I] \cdot \cos(\omega_t) & -[I] \cdot \sin(\omega_t) \\
[I] \cdot \sin(\omega_t) & [I] \cdot \cos(\omega_t) \\
\end{bmatrix},
\quad [I] = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 \\
\end{bmatrix}
\]

(4)

with

\[
\{x\} = [L] \cdot \{\xi\}
\]

(5)

\[
\{x\}' = [L]'' \cdot \{\xi\} + 2 \{L\}' \cdot \{\xi\}' + [L] \cdot \{\xi\}''
\]

(6)

This yields the transformed equation of motion

\[
[M^*] \cdot \{\xi\}'' + [[C^*] + [G^*]] \cdot \{\xi\}' + [K^*] \cdot \{\xi\} = [0]
\]

(8)

with

\[
[M^*] = [M],
\quad [C^*] = 2\vartheta \cdot [A],
\quad [K^*] = \begin{bmatrix}
-\rho \omega^2 + 1 + \kappa & -1 \\
-1 & -\omega^2 + 1 \\
\end{bmatrix},
\quad [G^*] = \begin{bmatrix}
0 & 0 & 0 & -2w \\
0 & 0 & 0 & -2w \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

The matrix \([M^*] = [M^*]^T \) is positive definite and the matrix \([C^*] = [C^*]^T \) is semi-definite. According to the above mentioned stability definition, the matrix \([K^*] \) should be positive definite. After some calculations, one gets as boundary frequency

\[
w_b = \sqrt{\Gamma - \sqrt{\Gamma^2 - \frac{\kappa}{\rho}}}
\]

(9)

with \( \Gamma = \frac{1}{2}(1 + \frac{1 + \kappa}{\rho}) \). The boundary frequency \( w_b \) coincide with the lower critical frequency of the system \( w_{cr1} \). The upper one is

\[
w_{cr11} = \sqrt{\Gamma + \sqrt{\Gamma^2 - \frac{\kappa}{\rho}}}.
\]

(10)
In figure 2 the influence of the stiffness ratio $\kappa = \frac{k_i}{k_R}$ and the mass ratio $\rho = \frac{m_i}{m_R}$ on the boundary frequency $w_b$ are shown. For a huge stiffness ratio $\kappa$, that means a stiff shaft in respect to the rotor shaft connection the boundary frequency converge to $w_b = 1$. In this case the boundary frequency is mainly affected by the internal spring stiffness $k_i$ and the mass of the rotor $m_R$. For a small mass ratio $\rho$, that means a huge rotor mass in respect to the shaft the boundary frequency converge faster to one for increasing stiffness ratio $\kappa$. If the stiffness ratio is small, e.g. an elastic shaft the boundary frequency becomes much smaller than one ($w_b \ll 1$).

By introducing outer damping $c_a$, the boundary frequency $w_b$ can be increased. In the following it is studied in which range the system can be stabilized.

The previously used approach, transforming the equation of motion such that the above mentioned stability condition of [1] can be adopted is not possible in this case. It is impossible to determine a suitable transformation matrix $[L]$ which fulfills equation (3) and the condition to be orthogonal at the same time. A matrix $[L]$ which solves equation (3) causes that the mass matrix $[M^*]$ and the stiffness matrix $[K^*]$ in the transformed equation become time-dependent.

It is possible to determine a stability range which coincide with the one of the system with no outer damping by using the stability definition of Liapunov. However it is not possible to find a Liapunov function, which is adequately close to show the influence of the outer damping.

Applying the Hurwitz criterion to identify the boundary condition is analytically extensive. Similarly the analytical calculation of the real part of the eigenvalues gets extensive or even impossible.

Hence the system with outer damping is evaluated numerically. The stiffness, mass and outer damping ratio and the internal damping are changed in a certain range during calculation such that 200 different configurations are computed. For each parameter set the speed of revolution is altered step-by-step from zero to a maximum speed. Once the real part of one eigenvalue becomes positive during calculation, the previous speed step is stored as boundary frequency $w_b$. The calculation was performed with FERAN. Some results are shown in the figures 3 and 4.

With increasing stiffness ratio, that means with a stiffer shaft or a smaller spring stiffness between rotor and shaft, it becomes more and more difficult to enlarge the stability range by increasing the outer damping $c_a$ (figure 3 and figure 4). An increased internal damping $c_i$, that means an increased internal damping ratio has a negative impact on the possibility to heighten the boundary frequency by altering the outer damping (figure 3 compared to figure 4).

Considering the simulation results it is possible to stabilize the system under certain conditions beyond the second critical speed by increasing the outer damping ratio $\varepsilon$. 
III. Example: Lab centrifuge with viscous internal damping

To illustrate modelling and simulation of viscous internal damping between rotor and shaft we revert to measurements which were undertaken at the Institute of Applied Mechanics in Clausthal. The stability range of a rotor system representing the upper part of a lab centrifuge was analyzed [6]. The rotor system consists of a rotor attached by a rubber element to the shaft (figure 5).

Fig. 5. Simulation model of a lab centrifuge with a rubber element as rotor shaft connection. Stiffness of the rubber element \( k_c \) and internal damping of the rubber element \( c_c \), outer damping \( c_a \).

The rubber element is specified by three rotating spring damper elements in the simulation model of FERAN, acting in translational direction. As three spring damper elements are used both tilting of the rotor and movements transverse to the shaft direction are covered. Hence the values for damping and stiffness in translational and rotational direction determined by measurements can be reflected in the simulation model.

The rotor is assumed to be rigid. Thus in FERAN nodes one to five are connected rigidly with constraints. The mass properties of the rotor are connected to node five. Its position is in the center of gravity of the rotor. Three sets were analyzed. Set two and three consists of the same rubber element which is less stiff than that of set one. Additional outer damping \( c_a \) was introduced onto the rotor of set two and three.

Table I contains the parameters for the rotor, like axial moment of inertia \( J_A \), polar moment of inertia \( J_P \) (both in respect to node five), mass \( m \), for the rubber element like stiffness \( k_c \) (translation), \( k_{ri} \) (rotation), damping \( c_{i} \) (translation), \( c_{ri} \) (rotation), and parameters for the outer damping \( c_a \), used to analyze the rotor system in FERAN. The shaft diameter varies between 22mm and 10mm for set one and has a total length of 52mm. The shaft of set two has a diameter between 22mm and 9mm and a total length of 50mm. The shaft of set three is identically to that of set two. It is obvious that the shaft of all sets is considerably stiffer than the rubber element which connects rotor and shaft.

Table I. Parameters used for the simulation of the lab centrifuge

<table>
<thead>
<tr>
<th>Set</th>
<th>( m [kg] )</th>
<th>( J_A [kgm^2] )</th>
<th>( J_P [kgm^2] )</th>
<th>( k_{ri} [N/m] )</th>
<th>( k_c [N/m] )</th>
<th>( c_{ri} [Ns/m] )</th>
<th>( c_a [Ns/m] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9.1</td>
<td>0.0185</td>
<td>0.0352</td>
<td>6.33E06</td>
<td>612</td>
<td>0.572</td>
<td>0.0226</td>
</tr>
<tr>
<td>2</td>
<td>9.1</td>
<td>0.0226</td>
<td>0.0355</td>
<td>8.3E06</td>
<td>3.93</td>
<td>0.051</td>
<td>0.0226</td>
</tr>
<tr>
<td>3</td>
<td>9.5</td>
<td>0.0226</td>
<td>0.0335</td>
<td>8.3E06</td>
<td>3.93</td>
<td>0.051</td>
<td>0.0226</td>
</tr>
</tbody>
</table>

As the outer damping due to material damping in the shaft of set one is negligible compared to the internal damping \( c_c \) the boundary frequency should coincide with the first resonance frequency \( f_r \). The simulation results and the measurements approve this assumption (table II). For frequencies of revolution higher than \( f = 105Hz \) respectively \( f = 108Hz \) instability is to expect.

The effects of outer damping are shown with set two and three. They are both identical besides the outer damping. The outer damping \( c_a \) of set three is higher than the one of set two.

For set two and set three the rotor rotates between 15Hz and 23Hz quiet turbulently since the first forward whirl is close to the frequency of revolution in this area. The first forward whirl is crossed in a small angel in the Campbell diagram. In this case uncertainties during the measurement of system parameters and during generation of the simulation model influence the position of the first critical speed in the simulation decisively. A resonance frequency of \( f_r = 18Hz \) was measured. The simulation yields a resonance frequency of \( f_r = 23Hz \).

With the help of outer damping \( c_a \) the rotor system can be stabilized above the resonance frequency. By increasing the outer damping like between set two and set three the boundary frequency moves upward. Both effects are reflected in the simulation and the measurements. At 39Hz (set two) respectively 50Hz (set three) huge rotor deflections were determined during the measurements such that the frequency of revolution was not increased above this values for safety reasons. The simulation yields a higher boundary frequency for both sets. However in the simulation results the damping of the critical first forward whirl is already re-
duced to a value below 1 % for a frequency of revolution of 39 Hz (set two) and 50 Hz (set three).

IV. Dry Friction in rotor shaft connections

If micro movements occur between rotor and shaft surface the energy dissipation can be described with dry friction. The friction force derogates the movement of the shaft fibres. Thus an additional tension and compression of the fibres occurs, which leads to an additional force vector destabilizing the system in the post-critical speed range.

Dry friction was investigated among others by [4]. They studied nonlinear forced oscillations of a rotating shaft with nonlinear spring characteristics and internal damping via measurements and simulation. [5] analyzed micro-slip and material damping in the rotor slot wedges of a turbine generator.

The effects of dry friction in a rotor shaft connection of a Jeffcott rotor are investigated in the following. We consider the pressure \( p \) in the joint between rotor and shaft as non-varying during operation and not dependent on the frequency of revolution. If we assume small shaft deflections \( \{d\} \), the pressure \( p \) acts as an uniform load \( p \cdot b_p \) on the shaft circumference, in the location of node 1 and node 3 (figure 6). Vector \( \{d\} = \{ v(t) \ w(t) \}^T \) consists of the displacements \( v(t) \) and \( w(t) \) transverse to the shaft direction.

![Fig. 6. Model of a Jeffcott rotor with dry friction acting between shaft and rotor](image)

The direction of the friction force depends on the tilting speed of the shaft diameter in the rotating coordinate system \( \{ \theta_i^* \} \). The friction moment of node \( i \) can be stated similar to [7] as

\[
\{ M_{bi|i}^* \} = -\kappa \cdot \frac{\{ \theta_i^* \}}{|\{ \theta_i^* \}|}.
\] (11)

To calculate \( \kappa \) we integrate the friction moment due to the load \( p \cdot b_p \) at node \( i \) over the circumference of the cross section. This yields:

\[
\kappa = 4 \cdot \mu \cdot p \cdot b_p \cdot r_w^2.
\] (12)

The friction moment of each node \( i \) is transformed into the reference coordinate system:

\[
\{ M_{bi|i} \} = -\frac{\kappa}{\Delta} \left\{ \dot{\theta}_i + \Omega \theta_i \right\}
\] (13)

with \( \Delta = \sqrt{(\dot{\theta}_i + \Omega \theta_i)^2 + (\dot{\theta}_j - \Omega \theta_j)^2} \).

Micro-movements occur when the axial force in the rotor shaft connection exceeds the friction force. To facilitate the model we suppose that the shaft movement is not affected by the rotors stiffness and geometry when rotor and shaft are connected rigidly. This assumption holds when the width of the joint is small in respect to the shaft length, and the shaft is slender. Furthermore the moment \( M_{j1} \) in the joint of rotor and shaft due to the shafts bending in node 1 (figure 6) is approximated by a function of the magnitude of rotor deflection \(|\{d\}|\) in the fixed coordinate system:

\[
M_{j1} = \frac{3 \cdot E \cdot |\{d\}| \cdot r_w^2 \cdot \pi}{l_g^2 \cdot (3/2)^2 - 3/8}.
\] (14)

This function is obtained applying Bernoulli beam theory. Now we can state the condition that micro movements appear:

\[
|M_{bi1}| < |M_{j1}|. \quad (15)
\]

Let the friction moment of node 1. To facilitate numerical integration, the two friction moments on node 1 and node 3 are replaced by an equivalent force vector acting on node 2 in figure 6.

In figure 7 the results of a run-up of this rotor system are shown. The magnitude of rotor displacement \(|\{d\}|\) in figure 7 is calculated by \( \sqrt{v(t)^2 + w(t)^2} \) with the displacements \( v(t) \) and \( w(t) \) transverse to the shaft direction. The numerical integration was done with a modified Rosenbrock method in the simulation program FERAN.

The dynamic simulation over the whole speed range is one way to show the effects of dry friction. After passing the first critical speed at 33 Hz the rotor oscillates due to dry friction with the frequency of the first forward whirl added by a vibration with the frequency of revolution. As the magnitude of rotor displacement is shown one can only observe the beat frequency in the post critical frequency area which is the difference between frequency of revolution and first forward whirl.

Both for the case with internal dry friction \( p > 0 \) N/mm² and for the case without internal dry friction \( p = 0 \) N/mm², the beat frequency is the same, as the frequency of the first forward whirl does not change with the frequency of revolution.

Increased pressure in the shaft rotor and consequently an increased friction moment leads to higher magnitudes in
the post critical frequency range provided that micro movements occur. In the sub critical speed range it is possible to figure out the point, were micro movements start to arise due to increasing rotor deflections as the condition of equation (15) is fulfilled. Additional vibrations can be observed. It is obvious that the frequency of revolution, above which micro movements start is higher if the pressure in the rotor shaft connection is higher.

The heightened amplitudes in the post critical speed range due to internal damping are a result of the reducing damping effects of rotating energy dissipation.

![Graph showing run-up of a Jeffcott rotor with internal dry friction between rotor and shaft.](image)

Fig. 7. Run-up of a Jeffcott rotor with internal dry friction between rotor and shaft.

V. Conclusion

Different approaches of modelling internal damping were shown. In respect to viscous internal damping in rotor shaft connections it was possible to deduce an analytical formulation for the boundary frequency for a simplified rotor system with no outer damping. This model is closer to practical applications when viscous internal damping between rotor and shaft is more decisive than the internal material damping in the shaft. The influence of system parameters when outer damping is applied was shown numerically. The stability boundaries for lab centrifuges can be computed quiet well with FERAN, a rotor dynamic simulation tool, where viscous internal damping is embedded. This was shown by a comparison between measurements and simulation results.

To describe micro movements and dry friction in the connection of shaft and rotor of lab centrifuges it is necessary to extend the simplified model of the Jeffcott rotor in section IV. Measurements are necessary to figure out the main influence parameters of dry frictions. Efforts in this direction are in preparation.

References
